HARVARD ENGINEERING SCHOOL

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59 S—15500-2-17-02-2.n

A TREATISE

ON

TELEGRAPHY

PREPARED FOR STUDENTS OF

THE INTERNATIONAL CORRESPONDENCE SCHOOLS

SCRANTON, PA.

Volume IV

ANSWERS TO QUESTIONS

First Edition

SCRANTON

INTERNATIONAL TEXTBOOK COMPANY

1901

1363

Engineering App.

b

BURR PRINTING HOUSE,
FRANKFORT AND JACOB STREETS,
NEW YORK.

NOTICE.

There is a break in the continuity of the question numbers and figure numbers between the answers to questions relating to the section on Elementary Algebra and Trigonometric Functions and those relating to the section on Elementary Mechanics, and between those relating to Elementary Mechanics and those relating to Principles of Electricity and Magnetism. This does not affect the subject matter, which is published in full, and in regular order.

A KEY

TO ALL THE

QUESTIONS AND EXAMPLES

INCLUDED IN THE

QUESTION PAPERS IN VOLS. I AND II.

This volume contains the Keys to the various Question Papers in Vols. I and II. These Keys have been so arranged as to be similar in all respects to the Question Papers to which they refer. The last seven Keys bear the same section numbers as the Question Papers in Vol. II.

CONTENTS.

ARITHMETIC.

(QUESTIONS 1-76.)

(1) See Art. **1.**

(2) See Art. **3.**

(3) See Arts. **5** and **6.**

(4) See Arts. **10** and **11.**

(5) 980 = Nine hundred eighty.

605 = Six hundred five.

28,284 = Twenty-eight thousand, two hundred eighty-four.

9,006,042 = Nine millions, six thousand and forty-two.

850,317,002 = Eight hundred fifty millions, three hundred seventeen thousand and two.

700,004 = Seven hundred thousand and four.

(6) Seven thousand six hundred = 7,600.

Eighty-one thousand, four hundred two = 81,402.

Five millions, four thousand and seven = 5,004,007.

One hundred and eight millions, ten thousand and one = 108,010,001.

Eighteen millions and six = 18,000,006.

Thirty thousand and ten = 30,010.

(7) See Art. **71.**

(8) See Art. **76.**

(9) See Art. **73.**

(10) See Art. **73.**

(11) See Art. **74.**

(12) $\frac{13}{8}$ is an improper fraction, since its numerator 13 is greater than its denominator 8.

(13) $4\frac{1}{2}$; $14\frac{3}{10}$; $85\frac{4}{19}$.

(14) To reduce a fraction to its lowest terms means to change its form without changing its value. In order to do this, we must divide both numerator and denominator by the same number until we can no longer find any number (except 1) which will divide both of these terms without a remainder.

To reduce the fraction $\frac{4}{8}$ to its lowest terms, we divide both numerator and denominator by 4, and obtain as a result the fraction $\frac{1}{2}$. Thus, $\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$; similarly, $\frac{4 \div 4}{16 \div 4} = \frac{1}{4}$; $\frac{8 \div 4}{32 \div 4} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$; $\frac{32 \div 8}{64 \div 8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$.

(15) When the denominator of any number is not expressed, it is understood to be 1, so that $\frac{6}{1}$ is the same as $6 \div 1$, or 6. To reduce $\frac{6}{1}$ to an improper fraction whose denominator is 4, we must multiply both numerator and denominator by some number which will make the denominator of 6 equal to 4. Since this denominator is 1, by multiplying both terms of $\frac{6}{1}$ by 4, we will have $\frac{6 \times 4}{1 \times 4} = \frac{24}{4}$, which has the *same value* as 6, but has a *different form*.

(16) In order to reduce a mixed number to an improper fraction, we must *multiply the whole number by the denominator of the fraction*, and *add the numerator of the fraction to that product*. This *result* is the *numerator of the improper fraction*, of which the *denominator is the denominator of the fractional part of the mixed number*.

$7\frac{7}{8}$ means the same as $7 + \frac{7}{8}$. In 1 there are $\frac{8}{8}$; hence, in

7 there are $7 \times \frac{8}{8} = \frac{56}{8}$. Add the $\frac{7}{8}$ of the mixed number and we obtain $\frac{56}{8} + \frac{7}{8} = \frac{63}{8}$, which is the required improper fraction.

$$13\frac{5}{16} = \frac{(13 \times 16) + 5}{16} = \frac{213}{16};\ 10\frac{3}{4} = \frac{(10 \times 4) + 3}{4} = \frac{43}{4}.$$

(17) The value of a fraction is obtained by dividing the numerator by the denominator.

To obtain the value of the fraction $\frac{13}{2}$ we divide the numerator 13 by the denominator 2. 2 is contained in 13 six times, with 1 remaining. This 1 remaining is written over the denominator 2, thereby making the fraction $\frac{1}{2}$, which is annexed to the whole number 6, and we obtain $6\frac{1}{2}$ as the mixed number. The reason for performing this operation is the following: In 1 there are $\frac{2}{2}$ (two halves), and in $\frac{13}{2}$ (thirteen halves) there are as many ones (1) as 2 is contained times in 13, which is 6, and $\frac{1}{2}$ (one-half) remaining. Hence, $\frac{13}{2} = 6 + \frac{1}{2} = 6\frac{1}{2}$, the required mixed number.

$$\frac{17}{4} = 4\frac{1}{4};\ \frac{69}{16} = 4\frac{5}{16};\ \frac{16}{8} = 2;\ \frac{67}{64} = 1\frac{3}{64}.$$

(18) A fraction is one or more of the equal parts of a unit, and is expressed by a numerator and a denominator, while a decimal fraction is a number of *tenths*, *hundredths*, *thousandths*, etc., of a unit, and is expressed by placing a period (.) called a decimal point, to the left of the figures of the number, and omitting the denominator.

(19) See Arts. **158** and **159.**

(20) To reduce the fraction $\frac{1}{2}$ to a decimal, we annex one cipher to the numerator, which makes it 1.0. Dividing 1.0, the numerator, by 2, the denominator, gives a quotient of .5, the decimal point being placed before the *one* figure of

the quotient, or .5, since only *one* cipher was annexed to the numerator.

$\frac{7}{8}$

```
8)7.000
  .875
```

$\frac{5}{32}$

```
32)5.00000(.15625
   32
   180
   160
    200
    192
     80
     64
     160
     160
```

$\frac{125}{1000}$

```
1000)125.000(.125
     1000
      2500
      2000
       5000
       5000
```

$\frac{65}{100}$

```
100)65.00(.65
    600
     500
     500
```

tenths. hundredths.

(21) . 0 8 = *Eight* **hundredths.**

tenths. hundredths. thousandths.

. 1 3 1 = *One hundred thirty-one* **thousandths.**

tenths. hundredths. thousandths. ten-thousandths.

. 0 0 0 1 = *One* **ten-thousandth.**

tenths. hundredths. thousandths. ten-thousandths. hundred-thousandths. millionths.

0 0 0 0 2 7 = *Twenty-seven* **millionths.**

tenths.	hundredths.	thousandths.	ten-thousandths.	
.0	1	0	8	= *One hundred eight* **ten-thousandths.**

	tenths.	hundredths.	thousandths.	ten-thousandths.	
93.	0	1	0	1	=Ninety-three, and *one hundred one* **ten-thousandths.**

In reading decimals, read the number just as you would if there were no ciphers before it. Then count from the decimal point towards the right, beginning with tenths, to as many places as there are figures, and the *name* of the last figure must be annexed to the previous reading of the figures to give the decimal reading. Thus, in the first example given, the simple reading of the figure is *eight*, and the name of its position in the decimal scale is **hundredths,** so that the decimal reading is *eight* **hundredths.** Similarly, the figures in the fourth example are ordinarily read *twenty-seven;* the name of the position of the figure 7 in the decimal scale is **millionths,** giving, therefore, the decimal reading as *twenty-seven* **millionths.**

If there should be a whole number before the decimal point, read it as you would read any whole number, and read the decimal as you would if the whole number were not there; or, read the whole number and then say "and" so many hundredths, thousandths, or whatever it may be, as "ninety-three, and *one* hundred one ten-thousandths."

(22) See Arts. **136** and **141.**

(23) See Art. **148.**

(24) See Art. **155.**

(25) In adding whole numbers, place the numbers to be added directly under each other, so that the extreme right-hand figures will stand in the same column, regardless of the position of those at the left. Add the first column of figures at the extreme right, which equals 19 units, or 1 ten and 9 units. We place 9 units under the units column and reserve 1 ten for the column of tens, $8 + 7 + 9 + 1 = 25$ tens, or 2 hundreds and 5 tens. Place 5 tens under the tens column, and reserve 2 hundreds for the hundreds column. $4 + 5 + 2 + 2 = 13$ hundreds, or 1 thousand and 3 hundreds. Place 3 hundreds under the hundreds column, and reserve the 1 thousand for the thousands column. $2 + 5 + 3 + 1 = 11$ thousands, or 1 ten-thousand and 1 thousand. Place the 1 thousand in the column of thousands, and reserve the 1 ten-thousand for the column of ten-thousands. $6 + 1 = 7$ ten-thousands. Place this seven ten-thousands in the ten-thousands column. There is but one figure, 8, in the hundreds of thousands place in the numbers to be added, so it is placed in the hundreds of thousands column of the sum.

$$\begin{array}{r} 3290 \\ 504 \\ 865403 \\ 2074 \\ 81 \\ 7 \\ \hline 871359 \end{array} \quad \text{Ans.}$$

A simpler (though less scientific) explanation of the same problem is the following : $7 + 1 + 4 + 3 + 4 + 0 = 19$; write the nine and reserve the 1. $8 + 7 + 0 + 0 + 9 + 1$ reserved $= 25$; write the 5 and reserve the 2. $0 + 4 + 5 + 2 + 2$ reserved $= 13$; write the 3 and reserve the 1. $2 + 5 + 3 + 1$ reserved $= 11$; write the 1 and reserve 1. $6 + 1$ reserved $= 7$; write the 7. Bring down the 8 to its place in the sum.

(26)

$$\begin{array}{r} 709 \\ 8304725 \\ 391 \\ 100302 \\ 300 \\ 909 \\ \hline 8,407,336 \end{array} \quad \text{Ans.}$$

(27) $\frac{1}{8}+\frac{2}{8}+\frac{5}{8}=\frac{1+2+5}{8}=\frac{8}{8}=1.$ Ans.

When the *denominators* of the fractions to be added *are alike*, we know that the units are divided into the *same number of parts* (in this case *eighths*); we, therefore, *add the numerators* of the fractions to find the number of parts (eighths) taken or considered, thereby obtaining $\frac{8}{8}$, or 1, as the sum.

(28) When the *denominators* are *not alike* we know that the units are divided into *unequal parts*, so before adding them we must find a common denominator for the denominators of all the fractions. Reduce the fractions to fractions having this common denominator, add the numerators, and write the sum over the common denominator.

In this case, the least common denominator, or the least number that will contain all the denominators, is 16; hence, we must reduce all of these fractions to 16ths and then add their numerators.

$\frac{1}{4}+\frac{3}{8}+\frac{5}{16}=?$ To reduce the fraction $\frac{1}{4}$ to a fraction having 16 for a denominator, we must multiply both terms of the fraction by some number which will make the denominator 16. This number evidently is 4, hence, $\frac{1\times 4}{4\times 4}=\frac{4}{16}$.

Similarly, both terms of the fraction $\frac{3}{8}$ must be multiplied by 2 to make the denominator 16, and we have $\frac{3\times 2}{8\times 2}=\frac{6}{16}$. The fractions now have a common denominator 16; hence, we find their sum by adding the numerators and placing their sum over the common denominator, thus: $\frac{4}{16}+\frac{6}{16}+\frac{5}{16}=\frac{4+6+5}{16}=\frac{15}{16}$. Ans.

(29) When mixed numbers and whole numbers are to be added, add the fractional parts of the mixed numbers

separately, and if the resulting fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction.

$42 + 31\frac{5}{8} + 9\frac{7}{16} = ?$ Reducing $\frac{5}{8}$ to a fraction having a denominator of 16, we have $\frac{5 \times 2}{8 \times 2} = \frac{10}{16}$. Adding the two fractional parts of the mixed numbers, we have $\frac{10}{16} + \frac{7}{16} = \frac{10 + 7}{16} = \frac{17}{16} = 1\frac{1}{16}$.

The problem now becomes $42 + 31 + 9 + 1\frac{1}{16} = ?$

Adding all the whole numbers and the number obtained from adding the fractional parts of the mixed numbers, we obtain $83\frac{1}{16}$ as their sum.

$$\begin{array}{r} 42 \\ 31 \\ 9 \\ 1\frac{1}{16} \\ \hline 83\frac{1}{16} \end{array} \text{ Ans.}$$

(30) $29\frac{3}{4} + 50\frac{5}{8} + 41 + 69\frac{3}{16} = ?$ $\frac{3}{4} = \frac{3 \times 4}{4 \times 4} = \frac{12}{16}$; $\frac{5}{8} = \frac{5 \times 2}{8 \times 2} = \frac{10}{16}$. $\frac{12}{16} + \frac{10}{16} + \frac{3}{16} = \frac{12 + 10 + 3}{16} = \frac{25}{16} = 1\frac{9}{16}$.

The problem now becomes $29 + 50 + 41 + 69 + 1\frac{9}{16} = ?$

$$\begin{array}{rl} 29 & \text{square inches.} \\ 50 & \text{square inches.} \\ 41 & \text{square inches.} \\ 69 & \text{square inches.} \\ 1\frac{9}{16} & \text{square inches.} \\ \hline 190\frac{9}{16} & \text{square inches.} \quad \textbf{Ans.} \end{array}$$

(31) In addition of decimals, the *decimal points must be placed directly under each other*, so that *tenths* will come *under tenths*, *hundredths under hundredths*, *thousandths under thousandths*, etc. The addition is then performed as in whole numbers, *the decimal point of the sum being placed directly under the decimal points above.*

$$\begin{array}{l} .125 \\ .7 \\ .089 \\ .4005 \\ .9 \\ .000027 \\ \hline 2.214527 \quad \text{Ans.} \end{array}$$

(32)

$$\begin{array}{r} 927.416 \\ 8.274 \\ 372.6 \\ 62.07938 \\ \hline 1370.36938 \end{array} \quad \text{Ans.}$$

(33)

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.
. 0	1	7			
. 2					
. 0	0	0	0	4	7
. 2	1	7	0	4	7

.217047 = *Two hundred and seventeen thousand and forty-seven* **millionths.** Ans.

(34) It will be more convenient to add these numbers together if we express $348\frac{3}{4}$ decimally, since the remaining numbers are decimals. $348\frac{3}{4}$ is equal to 348.75, since $\frac{3}{4} = .75$.

$$\begin{array}{l} 4)\,3.00\,(.75 \\ 2\,8 \\ \hline 20 \\ 20 \\ \hline \end{array}$$

Since we now have the different weights expressed decimally, we find the total weight of the four lengths of shafting to be 316.5 lb. + 402.3 lb. + 348.75 lb. + 309.4 lb., or 1,376.95 lb. Ans.

```
 316.5  lb.
 402.3  lb.
 348.75 lb.
 309.4  lb.
---------
1376.95 lb. Ans.
```

(35) If the cost of the coal consumed by a nest of steam boilers amounts to $15.83 on Monday, to $14.70 on Tuesday, to $14.28 on Wednesday, to $13.87 on Thursday, to $14.98 on Friday, and to $12.65 on Saturday, then, we find the total cost of the week's supply by adding the different amounts together; hence, $15.83 + $14.70 + $14.28 + $13.87 + $14.98 + $12.65 = $86.31.

```
$15.83
 14.70
 14.28
 13.87
 14.98
 12.65
------
$86.31 Ans.
```

(36) The steam engine, during the 12-hour test, showed that the number of revolutions made were 150,508, since 12,600 + 12,444 + 12,467 + 12,528 + 12,468 + 12,590 + 12,610 + 12,589 + 12,576 + 12,558 + 12,546 + 12,532 = 150,508 rev. Ans.

```
 12600 revolutions.
 12444 revolutions.
 12467 revolutions.
 12528 revolutions.
 12468 revolutions.
 12590 revolutions.
 12610 revolutions.
 12589 revolutions.
 12576 revolutions.
 12558 revolutions.
 12546 revolutions.
 12532 revolutions.
------
150508 revolutions. Ans.
```

(37) In subtracting whole numbers, place the subtrahend, or smaller number, under the minuend, or larger number, so that the right-hand figures stand directly under each other. Begin *at the right* to subtract.

$$\begin{array}{lr} (a) & 50962 \\ & 3338 \\ \hline & 47624 \end{array} \text{ Ans.}$$

We can not subtract 8 units from 2 units, so we take 1 ten from the 6 tens and add it to the 2 units. 1 *ten* = 10 *units*, so we have 10 units + 2 units = 12 units. Then, 8 units from 12 units leaves 4 units. We took 1 ten from 6 tens, so only 5 tens remain. 3 tens from 5 tens leaves 2 tens. In the hundreds column we have 3 hundreds from 9 hundreds leaves 6 hundreds. We can not subtract 3 thousands from 0 thousands, so we take 1 ten-thousand from 5 ten-thousands and add it to the 0 thousands. 1 *ten-thousand* = 10 *thousands*, and 10 thousands + 0 thousands = 10 thousands. Subtracting, we have 3 thousands from 10 thousands leaves 7 thousands. We took 1 ten-thousand from 5 ten-thousands and have 4 ten-thousands remaining. Since there are no ten-thousands in the subtrahend, the 4 in the ten-thousands column in the minuend is brought down into the same column in the remainder, because 0 from 4 leaves 4.

$$\begin{array}{lr} (b) & 15339 \\ & 10001 \\ \hline & 5338 \end{array} \text{ Ans.}$$

(38) $$\begin{array}{lr} (a) & 70968 \\ & 32975 \\ \hline & 37993 \end{array} \text{ Ans.} \qquad \begin{array}{lr} (b) & 100000 \\ & 98735 \\ \hline & 1265 \end{array} \text{ Ans.}$$

(39) $\frac{7}{8} - \frac{7}{16} = ?$ When the *denominators* of fractions are *not alike*, it is evident that the units are divided into *unequal parts*, therefore, before subtracting, *reduce the fractions to fractions having a common denominator*. Then, *subtract the numerators, and place the remainder over the common denominator.*

$$\frac{7 \times 2}{8 \times 2} = \frac{14}{16}. \quad \frac{14}{16} - \frac{7}{16} = \frac{14 - 7}{16} = \frac{7}{16}. \quad \text{Ans.}$$

$13 - 7\frac{7}{16} = ?$ This problem may be solved in two ways:

First: $13 = 12\frac{16}{16}$, since $\frac{16}{16} = 1$, and $12\frac{16}{16} = 12 + \frac{16}{16} = 12 + 1 = 13.$

$$\begin{array}{r} 12\frac{16}{16} \\ 7\frac{7}{16} \\ \hline 5\frac{9}{16} \end{array}$$

We can now subtract the whole numbers separately, and the fractions separately, and obtain $12 - 7 = 5$ and $\frac{16}{16} - \frac{7}{16} = \frac{16 - 7}{16} = \frac{9}{16}$. $5 + \frac{9}{16} = 5\frac{9}{16}$. Ans.

Second: By reducing both numbers to improper fractions having a denominator of 16.

$$13 = \frac{13}{1} = \frac{13 \times 16}{1 \times 16} = \frac{208}{16}. \quad 7\frac{7}{16} = \frac{(7 \times 16) + 7}{16} = \frac{112 + 7}{16} = \frac{119}{16}.$$

Subtracting, we have $\frac{208}{16} - \frac{119}{16} = \frac{208 - 119}{16} = \frac{89}{16}$ and

$$\frac{89}{16} = \begin{array}{r} 16)89(5\frac{9}{16} \\ 80 \\ \hline 9 \\ \hline 16 \end{array}$$

the same result that was obtained by the first method.

$312\frac{9}{16} - 229\frac{5}{32} = ?$ We first reduce the fractions of the two mixed numbers to fractions having a common denominator. Doing this, we have $\frac{9}{16} = \frac{9 \times 2}{16 \times 2} = \frac{18}{32}$. We can now subtract the whole numbers and fractions separately, and have $312 - 229 = 83$ and $\frac{18}{32} - \frac{5}{32} = \frac{18 - 5}{32} = \frac{13}{32}$.

$$\begin{array}{r} 312\frac{18}{32} \\ 229\frac{5}{32} \\ \hline 83\frac{13}{32} \end{array} \quad 83 + \frac{13}{32} = 83\frac{13}{32}. \quad \text{Ans.}$$

(40) (*a*) In subtraction of decimals, *place the decimal points directly under each other*, and proceed as in the subtraction of whole numbers, placing the *decimal point in the remainder directly under the decimal points above.*

$$\begin{array}{r} 709.6300 \\ .8514 \\ \hline 708.7786 \end{array} \quad \text{Ans.}$$

In the above example, we proceed as follows: We can not

subtract 4 ten-thousandths from 0 ten-thousandths, and as there are no thousandths, we take 1 hundredth from the three hundredths. 1 *hundredth* = 10 *thousandths* = 100 *ten-thousandths.* 4 ten-thousandths from 100 ten-thousandths leaves 96 ten-thousandths. 96 ten-thousandths = 9 *thousandths* + 6 *ten-thousandths.* Write the 6 ten-thousandths in the ten-thousandths place in the remainder. The next figure in the subtrahend is 1 thousandth. This must be subtracted from the 9 thousandths which is a part of the 1 hundredth taken previously from the 3 hundredths. Subtracting, we have 1 thousandth from 9 thousandths leaves 8 thousandths, the 8 being written in its place in the remainder. Next we have to subtract 5 hundredths from 2 hundredths (1 hundredth having been taken from the 3 hundredths leaves but 2 hundredths now.) Since we can not do this, we take 1 tenth from 6 tenths. 1 tenth = 10 hundredths, and 10 hundredths + 2 hundredths = 12 hundredths. 5 hundredths from 12 hundredths leaves 7 hundredths. Write the 7 in the hundredths place in the remainder. Next we have to subtract 8 tenths from 5 tenths (5 tenths now, because 1 tenth was taken from the 6 tenths). Since this can not be done, we take 1 unit from the 9 units. 1 *unit* = 10 *tenths.* 10 tenths + 5 tenths = 15 tenths, and 8 tenths from 15 tenths leaves 7 tenths. Write the 7 in the tenths place in the remainder. In the minuend we now have 708 units (one unit having been taken away) and 0 units in the subtrahend. 0 units from 708 units leaves 708 units, hence, we write 708 in the remainder.

$$\text{(b)}\quad \begin{array}{r} 81.963 \\ 1.700 \\ \hline 80.263 \end{array}\ \text{Ans.} \qquad \text{(c)}\quad \begin{array}{r} 18.00 \\ .18 \\ \hline 17.82 \end{array}\ \text{Ans.} \qquad \text{(d)}\quad \begin{array}{r} 1.000 \\ .001 \\ \hline .999 \end{array}\ \text{Ans.}$$

(*e*) $872.1 - (.8721 + .008) = ?$ In this problem we are to subtract $(.8721 + .008)$ from 872.1. First perform the operation as indicated by the sign between the decimals enclosed by the parenthesis.

$$\begin{array}{r} .8721 \\ .0080 \\ \hline .8801 \end{array}\ \textit{sum.}$$

Subtracting the sum (obtained by adding the decimals enclosed within the parenthesis) from the number 872.1 (as required by the minus sign before the parenthesis), we obtain the required remainder.

$$\begin{array}{r} 872.1000 \\ .8801 \\ \hline 871.2199 \end{array} \text{ Ans.}$$

(*f*) $(5.028 + .0073) - (6.704 - 2.38) = ?$ First perform the operations as indicated by the signs between the numbers enclosed by the parentheses. The first parenthesis shows that 5.028 and .0073 are to be added. This gives 5.0353 as their sum.

$$\begin{array}{r} 5.0280 \\ .0073 \\ \hline 5.0353 \end{array} \textit{ sum.}$$

$$\begin{array}{r} 6.704 \\ 2.380 \\ \hline 4.324 \end{array} \textit{ diff.}$$

The second parenthesis shows that 2.38 is to be subtracted from 6.704. The difference is found to be 4.324. The sign between the parentheses indicates that the quantities obtained by performing the above operations are to be subtracted, namely, that 4.324 is to be subtracted from 5.0353. Performing this operation, we obtain .7113 as the final result.

$$\begin{array}{r} 5.0353 \\ 4.3240 \\ \hline .7113 \end{array} \text{ Ans.}$$

(**41**) In subtracting a decimal from a fraction, or subtracting a fraction from a decimal, either reduce the fraction to a decimal before subtracting, or reduce the decimal to a fraction and then subtract.

(*a*) $\frac{7}{8} - .807 = ?$ $\frac{7}{8}$ reduced to a decimal becomes

$$\begin{array}{r} 7 \\ 8\overline{)7.000} \\ .875 \end{array}$$

$$\begin{array}{r} .875 \\ .807 \\ \hline .068 \end{array} \text{ Ans.}$$

Subtracting .807 from .875 the remainder is .068, as shown.

(*b*) $.875 - \frac{3}{8} = ?$ Reducing .875 to a fraction we have $.875 = \frac{875}{1,000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; hence, $\frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8} = \frac{1}{2}$ or .5. **Ans.**

Or, by reducing $\frac{3}{8}$ to a decimal, $\frac{3}{8}\overline{)3.000}$ and then sub-
$.375$

tracting, we obtain $.875 - .375 = .5 = \frac{5}{10} = \frac{1}{2}$, the same answer as above.

$$\begin{array}{r} .875 \\ .375 \\ \hline .500 \end{array} \text{ Ans.}$$

(*c*) $\left(\frac{5}{32} + .435\right) - \left(\frac{21}{100} - .07\right) = ?$ We first perform the operations as indicated by the signs between the numbers enclosed by the parentheses. Reducing $\frac{5}{32}$ to a decimal, we obtain $\frac{5}{32} = .15625$ (see example 20).

Adding .15625 and .435, $\frac{21}{100} = .21$; subtracting,

$$\begin{array}{l} .15625 \\ .435 \\ \hline .59125 \end{array} \qquad\qquad \begin{array}{l} .21 \\ .07 \\ \hline .14 \textit{ diff.} \end{array}$$

We are now prepared to perform the operation indicated by the minus sign between the parentheses, which is,

$$\begin{array}{l} .59125 \\ .14 \\ \hline .45125 \textit{ diff.} \end{array}$$

Ans.

(*d*) This problem means that 33 millionths and 17 thousandths are to be added. Also, that 53 hundredths and 274 thousandths are to be added, and the smaller of these sums is to be subtracted from the larger sum. Thus, $(.53 + .274) - (.000033 + .017) = ?$

tenths.	hundredths.	thousandths.
.5	3	
.2	7	4
.8	0	4 *sum.*

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.
.0	0	0	0	3	3
.0	1	7			
.0	1	7	0	3	3 *sum.*

$$\begin{array}{ll} .804 & \textit{larger sum.} \\ .017033 & \textit{smaller sum.} \\ \hline .786967 & \textit{diff.} \end{array}$$

Ans.

(42) $482\frac{4}{5} + 316\frac{1}{3} + 390\frac{3}{4} =$ what ?

When mixed numbers are to be added, add the fractional parts of the mixed numbers separately, and, if the resulting

fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction. First, we will reduce the fractional parts $\frac{4}{5}$, $\frac{1}{3}$, and $\frac{3}{4}$ to equivalent fractions having the least common denominator. In this case the least common denominator equals the product of the denominators 5, 3, and 4, since we can not divide any two of them by any number (except 1) without having a remainder, as can be done in the examples in Art. **96.** Hence, the least common denominator $= 5 \times 3 \times 4 = 60$. Reducing $\frac{4}{5}$, $\frac{1}{3}$, and $\frac{3}{4}$ to fractions having this least common denominator, we have 60 divided by the first denominator 5, equals 12. Then, $\frac{4}{5} \times \frac{12}{12} = \frac{48}{60}$. 60 divided by the second denominator 3, equals 20. Then, $\frac{1}{3} \times \frac{20}{20} = \frac{20}{60}$. 60 divided by the third denominator 4, equals 15. Then, $\frac{3}{4} \times \frac{15}{15} = \frac{45}{60}$. The sum of these fractions equals

$$\frac{48}{60} + \frac{20}{60} + \frac{45}{60} = \frac{48 + 20 + 45}{60} = \frac{113}{60}, \text{ or } 1\frac{53}{60}.$$

The problem now becomes $482 + 316 + 390 + 1\frac{53}{60}$, the sum of which equals $1{,}189\frac{53}{60}$.

$$\begin{array}{r} 482 \\ 316 \\ 390 \\ 1\frac{53}{60} \\ \hline 1189\frac{53}{60} \end{array}$$

1500 represents the actual horsepower required.

$1189\frac{53}{60}$ represents the indicated horsepower of the engines in use.

$310\frac{7}{60}$, or $310.11\frac{2}{3}$ = the H. P. to be developed by the new engine. Ans.

$\frac{7}{60}$ reduced to its equivalent decimal $= \frac{7}{60}$

$$
\begin{array}{r|l}
60 & 7.00(.11\frac{2}{3} \\
 & \underline{6\,0} \\
 & 1\,0\,0 \\
 & \underline{\ \ 6\,0} \\
 & \frac{4\cancel{0}}{6\cancel{0}} = \frac{2}{3}.
\end{array}
$$

(43)

3040 = No. of gallons in the tank at the beginning of the day.
4780 = No. of gallons pumped in during the morning.
7820 = No. of gallons in the tank after 4,780 gallons were added.
7240 = No. of gallons drawn out during the morning.
580 = No. of gallons in the tank at the beginning of the afternoon.
8675 = No. of gallons pumped in during the afternoon.
9255 = No. of gallons in the tank after 8,675 gallons were added.
7633 = No. of gallons drawn out during the afternoon.
1622 = No. of gallons remaining in the tank at night. Ans.

(44) The height of the stack equals 45 feet. Since six of the plates which make this stack are 7 feet long, the length of the six plates equals 6×7 or 42 feet. However, there is an allowance of 15 inches, or 1 foot 3 inches, for lapping, which, deducted from 42 feet, makes a total length of 40 feet 9 inches. Since the total height of the stack is 45 feet, the length of the seventh plate must equal 45 feet— 40 feet 9 inches, or 4 ft. 3 in. Ans.

45 feet 0 inches
40 feet 9 inches
4 feet 3 inches

(45) Since the inside diameter of the steam pipe is 6.06 inches, and the outside diameter is 6.62 inches, there is a difference of 6.62 — 6.06, or. 56 of an inch, in both diameters.

But .56 of an inch is just twice the thickness of the pipe; hence, the pipe is $\frac{1}{2}$ of .56, or .28 of an inch thick.

(46) In the multiplication of whole numbers, place the multiplier under the multiplicand, and multiply each term of the multiplicand by each term of the multiplier, writing the right-hand figure of each product obtained under the term of the multiplier which produces it.

(*a*)
$$\begin{array}{r} 526{,}387 \\ 7 \\ \hline 3{,}684{,}709 \end{array} \quad \text{Ans.}$$

7 times 7 units = 49 units, or 4 tens and 9 units. We write the 9 units and reserve the 4 tens. 7 times 8 tens = 56 tens; 56 + 4 tens reserved = 60 tens, or 6 hundreds and 0 tens. Write the 0 tens and reserve the 6 hundreds. 7 × 3 hundreds = 21 hundreds; 21 + 6 hundreds reserved = 27 hundreds, or 2 thousands and 7 hundreds. Write the 7 hundreds and reserve the 2 thousands. 7 × 6 thousands = 42 thousands; 42 + 2 thousands reserved = 44 thousands, or 4 ten-thousands and 4 thousands. Write the 4 thousands and reserve the 4 ten thousands. 7 × 2 ten-thousands = 14 ten-thousands; 14 + 4 ten-thousands reserved = 18 ten-thousands, or 1 hundred-thousand and 8 ten-thousands. Write the 8 ten-thousands and reserve the 1 hundred-thousand. 7 × 5 hundred-thousands = 35 hundred-thousands; 35 + 1 hundred-thousand reserved = 36 hundred-thousands. Since there are no more figures in the multiplicand to be multiplied, we write the 36 hundred-thousands in the product. This completes the multiplication.

A simpler (though less scientific) explanation of the same problem is the following:

7 times 7 = 49; write the 9 and reserve the 4. 7 times 8 = 56; 56 + 4 reserved = 60; write the 0 and reserve the 6. 7 times 3 = 21; 21 + 6 reserved = 27; write the 7 and reserve the 2. 7 × 6 = 42; 42 + 2 reserved = 44; write the 4 and reserve 4. 7 × 2 = 14; 14 + 4 reserved = 18; write the 8 and reserve 1. 7 × 5 = 35; 35 + 1 reserved = 36; write the 36.

(*b*)	700,298
	17
	4902086
	700298
	11,905,066 Ans.

In this case the multiplier is 17 *units*, or 1 *ten* and 7 *units*, so that the product is obtained by adding two partial products, namely, $7 \times 700,298$ and $10 \times 700,298$. The actual operation is performed as follows:

7 times $8 = 56$; write the 6 and reserve the 5. 7 times $9 = 63$; $63 + 5$ reserved $= 68$; write the 8 and reserve the 6. 7 times $2 = 14$; $14 + 6$ reserved $= 20$; write the 0 and reserve the 2. 7 times $0 = 0$; $0 + 2$ reserved $= 2$; write the 2. 7 times $0 = 0$; $0 + 0$ reserved $= 0$; write the 0. 7 times $7 = 49$; $49 + 0$ reserved $= 49$; write the 49.

To multiply by the 1 ten we have 1 ten times 8 units = 8 tens and 0 units. We do not write the 0 units, but write the 8 tens under the 8 tens in the first partial product above. We next multiply 1 ten, and 9 tens = 90 tens = 9 hundreds + 0 tens; write the 9 hundreds under the 0 hundreds of the first partial product above. Again, 1 ten times 2 hundreds = 2 thousands; write the 2 thousands under the 2 thousands above. 1 ten times 0 thousands = 0 ten-thousands; write the 0 in the ten-thousands place. 1 ten times 0 ten-thousands = 0 hundred-thousands; write the 0 in the hundred-thousands place. 1 ten times 7 hundred-thousands = 7 millions; write the 7 in the millions place. This completes the second partial product. Add the two partial products; their sum equals the entire product.

(*c*)	217
	103
	651
	000
	217
	22351
	67
	156457
	134106
	1,497,517 Ans

Multiply any two of the numbers together, and multiply their product by the third number.

(47) If your watch ticks every second, to find how many times it ticks in one week it is necessary to find the number of seconds in 1 week.

```
      60 seconds = 1 minute.
      60 minutes = 1 hour.
   ------
   3,600 seconds = 1 hour.
      24 hours = 1 day.
   ------
   14400
   7200
   ------
  86,400 seconds = 1 day.
       7 days = 1 week.
 -------
 604,800 seconds in one week, or the number of times that
   Ans.    your watch ticks in a week.
```

(48) (*a*) There are 3 decimal places in the multiplicand and 3 in the multiplier; hence, there are 3 + 3 or 6 decimal places in the product. Since the product contains but four figures, we prefix two ciphers in order to obtain the necessary six decimal places.

```
    .107
    .013
    ----
     321
    107
 -------
 .001391  Ans.
```

There are 2 decimal places in the multiplier and none in the multiplicand; hence, there are 2 + 0 or 2 decimal places in the first product.

Since in the second multiplication there are 2 decimal places in the multiplicand and 3 decimal places in the multiplier, there are 3 + 2 or 5 decimal places in the second product.

When there are one or more ciphers in the multiplier, multiply just the same as with the other figures.

```
(b)  203
     2.03
     ----
      609
     000
    406
   ------
   412.09
     .203
  -------
   123627
   00000
  82418
 --------
 83.65427  Ans.
```

(*c*) First perform the operations indicated by the signs between the numbers enclosed by the parentheses, and then whatever may be required by the sign between the parentheses.

```
 31.85
   2.7
 -----
 22295
 6370
 -----
 85.995
```

The first parenthesis shows that the numbers 2.7 and 31.85 are to be multiplied together.

The second parenthesis shows that 316 is to be taken from 3.16.

```
 3.160
  .316
 -----
 2.844
```

The product obtained by performing the operation indicated by the signs within the first parenthesis is now multiplied by the remainder obtained by performing the operation indicated by the signs within the second parenthesis.

```
     85.995
      2.844
 ----------
     343980
    343980
   687960
  171990
 ----------
 244.569780   Ans.
```

(*d*) $(107.8 + 6.541 - 31.96) \times 1.742 = ?$

```
    107.8              82.381
 +    6.541         ×   1.742
 ---------          ---------
    114.341            164762
 -   31.96            329524
 ---------           576667
     82.381          82381
                   ----------
                   143.507702   Ans.
```

(49) If an engine and boiler are worth $3,246, and the building is worth 3 times as much plus $1,200, then the building is worth

```
       $3246
           3 times
       -----
        9738
 plus   1200
       ------
      $10938 = value of building.
```

If the tools are worth twice as much as the building, plus \$1,875, then the tools are worth

$$\begin{array}{rr}
 & \$10938 \\
 & 2 \\
\hline
 & 21876 \\
\text{plus} & 1875 \\
\hline
 & \$23751 = \text{value of tools.}
\end{array}$$

$$\begin{array}{lr}
\text{Value of building} = & \$10938 \\
\text{Value of tools} \quad = & 23751 \\
\hline
 & \$34689 = \text{value of the building and tools.}
\end{array}$$

(*a*) Ans.

$$\begin{array}{lr}
\text{Value of engine and boiler} = & \$3246 \\
\text{Value of building and tools} = & 34689 \\
\hline
 & \$37935 = \text{value of the whole plant.}
\end{array}$$

(*b*) Ans.

(50) If one 3-inch tube measures $15\frac{1}{2}$ ft. in length, 60 of these tubes would measure $60 \times 15\frac{1}{2}$ ft., or 930 ft. in length. If 1 foot of tubing has a heating surface of .728 sq. ft., then it is evident that 930 ft. of tubing would have a heating surface of $930 \times .728$ ft., or 677.04 sq. ft.

$$\begin{array}{r}
15\frac{1}{2} \\
60 \\
\hline
900 \\
30 \\
\hline
930 \text{ ft.}
\end{array}$$

$$\begin{array}{r}
.728 \\
930 \\
\hline
21840 \\
6552 \\
\hline
677.040 \text{ sq. ft.}
\end{array}$$

(51) If in 1 hour 10 pounds of coal are burned for every square foot of grate area, and 9 pounds of water are evaporated for every pound of coal burned, then in 1 hour there would be 9×10 or 90 pounds of water evaporated for 1 sq. ft. of grate area; and since the grate area is 30 sq. ft., the amount of water evaporated would be $30 \times 90 =$ 2,700 lb. Since 2,700 lb. of water are evaporated in 1

hour, in a day of 10 hours there would be $10 \times 2{,}700$ lb., or 27,000 lb. of water evaporated.

(52) Each stroke of the engine is 18 inches in length. Since the piston makes 2 strokes for each revolution, it would pass over a distance of 2×18 inches $= 36$ inches, or 3 feet in 1 revolution, and in making 160 revolutions it would pass over 160×3, or 480 ft. Since 480 ft. are passed over in 1 minute, in 1 hour, or 60 minutes, the distance passed over equals $60 \times 480 = 28{,}800$ ft. Since the steam engine runs 6 days a week and $8\frac{1}{2}$ hours per day, the total number of hours it runs per week $= 6 \times 8\frac{1}{2}$, or 51 hours. If the piston passes over a distance of 28,800 ft. in 1 hour, in 51 hours it would pass over $51 \times 28{,}800$ ft., or 1,468,800 ft. Ans.

$$\begin{array}{r} 480\text{ ft.} \\ \times\ 60 \\ \hline 28800\text{ ft.} \end{array}$$

$$\begin{array}{r} 28800 \\ \times\ \ 51 \\ \hline 28800 \\ 144000 \\ \hline 1468800 \end{array}$$

(53) Since the first pump delivers $1\frac{1}{4}$ gallons for each stroke, and runs at the rate of 75 strokes per minute, then the number of gallons delivered by the first pump equals $75 \times 1\frac{1}{4} = 75 \times 1.25 = 93.75$ gallons.

$$\begin{array}{r} 1.25 \\ \times\ \ 75 \\ \hline 625 \\ 875 \\ \hline 93.75 \end{array}$$

$\frac{7}{8} = .875$, since

$$\begin{array}{r} 8\overline{)7.000} \\ \hline .875 \end{array}$$

The second pump delivers $\frac{7}{8}$ or .875 of a gallon for each stroke, and runs at the rate of 115 strokes per minute; then the total number of gallons delivered by the second pump equals $115 \times .875 = 100.625$ gallons.

$$\begin{array}{r} .875 \\ \times\ 115 \\ \hline 4375 \\ 875 \\ 875 \\ \hline 100.625 \end{array}$$

```
8)5.000
  .625

  1.625
×    96
  9750
 14625
156.000

 350.375
      60
21022.500
```

The third pump delivers $1\frac{5}{8}$, or 1.625 (since $\frac{5}{8} = .625$) gallons, for each stroke, and runs at the rate of 96 strokes per minute; then, the total number of gallons delivered by the third pump equals $96 \times 1.625 = 156$ gallons.

The total number of gallons delivered by the three pumps in one minute equals $93.75 + 100.625 + 156$ gallons, or 350.375 gallons. Since there are 60 minutes in 1 hour, we find that the total number of gallons of water fed to the boilers in 1 hour equals $60 \times 350.375 = 21{,}022.5$. Ans.

(54) (*a*)

```
84)962842.0000(11462.4047+ Ans.
   84
   122
    84
    388
    336
     524
     504
      202
      168
       340
       336
        400
        336
         640
         588
          52
```

84 is contained in 96, once. Place 1 as the first figure in the quotient and multiply the divisor 84 by it. Subtract

the product, which is 84, from 96, leaving a remainder of 12. Bring down the next figure in the dividend, which is 2, annex it to 12, making a new dividend of 122.

84 is contained in 122 once. Place 1 as the second figure in the quotient and multiply the divisor 84 by it. Subtract the product (84) from 122, leaving a remainder of 38. Bring down the next figure in the dividend, which is 8, annex it to 38, making a new dividend of 388.

84 is contained in 388 4 times. Place 4 as the third figure in the quotient and multiply the divisor 84 by it. The product is 336. Subtract the product from 388, leaving a remainder of 52. Bring down the next figure, which is 4, annex it to 52, making a new dividend of 524.

84 is contained in 524 6 times. Place 6 as the fourth figure in the quotient. Multiply the divisor 84 by it and subtract the product (504) from 524, leaving a remainder of 20. Bring down the next figure, which is 2, annex it to 20. making a new dividend of 202.

84 is contained in 202 2 times. Place 2 as the fifth figure in the quotient. Multiply the divisor 84 by it, and subtract the product (168) from 202, leaving a remainder of 34. If it is desired to carry the quotient to 4 decimal places, annex 4 ciphers to the dividend and continue in the same way. In the quotient point off as many decimal places as there are ciphers annexed, or, in this case, four decimal places.

```
(b) 63)39728.000(630.603+ Ans.
       378
       ---
        192
        189
        -----
          380*
          378
          ----
            200
            189
            ---
             11
```

*63 is not contained in 38, so we place a cipher in the quotient and bring down the next figure in the dividend, which is a cipher that has been annexed. making a new dividend of 380.

63 is contained in 380 6 times. $6 \times 63 = 378$. Subtracting

378 from 380 leaves 2. Bringing down the next figure in the dividend we have 20 for a new dividend. 63 is not contained in 20, so we place a cipher in the quotient and bring down the next cipher in the dividend, making a new dividend of 200. 63 is contained 3 times in 200.

```
(c) 108)29714.0000(275.1296  Ans.
        216
        ---
         811
         756
         ---
          554
          540
          ---
           140
           108
           ---
            320
            216
            ---
            1040
             972
             ---
              680
              648
              ---
               32
```

```
(d) 135)406089.0000(3008.0666
        405                  Ans.
        ---
        *1089
         1080
         ----
           900
           810
           ---
            900
            810
            ---
             900
             810
             ---
              90
```

*135 is not contained in 10, so we place a cipher as the second figure in the quotient. Bringing down the next figure 8 and annexing it to 10, we have a new dividend of 108. 135 is not contained in 108, so we place a cipher as the third figure in the quotient, and bring down the next figure in the dividend, or 9, and annex it to 108, making a new dividend of 1,089. 135 is contained in 1,089 8 times. Write 8 as the fourth figure in the quotient. Multiply 135 by 8 and subtract the product (1,080) from 1,089, which leaves a remainder of 9. Bring down the next figure in the dividend, which is a cipher that has been annexed, annex it to the remainder 9, making a new dividend of 90. As 135 is not contained in 90, we place a 0 in the quotient and bring down another cipher from the dividend, making a new dividend of 900. 135 is contained in 900 6 times.

Write 6 as the next figure in the quotient, multiply 135 by 6 and subtract the product (810) from 900, which leaves a remainder of 90. Bring down the next figure (0) in the dividend, annex it to the remainder 90, making a new dividend of 900. 135 is contained in 900 six times. Place 6 as the next figure in the quotient and multiply the divisor by it. It is plain that each succeeding figure of the quotient will be 6. Point off *four* decimal places in the quotient, since four ciphers were annexed.

(55) In division of fractions, ***invert the divisor*** (or, in other words, turn it upside down) ***and proceed as in multiplication.***

(*a*) $35 \div \frac{5}{16} = \frac{35}{1} \times \frac{16}{5} = \frac{35 \times 16}{1 \times 5} = \frac{560}{5} = 112.$ Ans.

(*b*) $\frac{9}{16} \div 3 = \frac{9}{16} \div \frac{3}{1} = \frac{9}{16} \times \frac{1}{3} = \frac{9 \times 1}{16 \times 3} = \frac{9}{48} = \frac{3}{16}.$ Ans.

(*c*) $\frac{17}{2} \div 9 = \frac{17}{2} \div \frac{9}{1} = \frac{17}{2} \times \frac{1}{9} = \frac{17 \times 1}{2 \times 9} = \frac{17}{18}.$ Ans.

(*d*) $\frac{113}{64} \div \frac{7}{16} = \frac{113}{64} \times \frac{16}{7} = \frac{113 \times 16}{64 \times 7} = \frac{1,808}{448} =$

$$\frac{452}{112} = \frac{113}{28}) \; 113 \; (4\tfrac{1}{28}. \quad \text{Ans.}$$

$$\begin{array}{r} 112 \\ \hline 1 \end{array}$$

(*e*) $15\frac{3}{4} \div 4\frac{3}{8} = ?$ Before proceeding with the division, reduce both of the mixed numbers to improper fractions. Thus, $15\frac{3}{4} = \frac{(15 \times 4)+3}{4} = \frac{60+3}{4} = \frac{63}{4}$, and $4\frac{3}{8} = \frac{(4 \times 8)+3}{8} = \frac{32+3}{8} = \frac{35}{8}.$ The problem is now $\frac{63}{4} \div \frac{35}{8} = ?$ As before, invert the divisor and multiply; $\frac{63}{4} \div \frac{35}{8} = \frac{63}{4} \times \frac{8}{35} = \frac{63 \times 8}{4 \times 35} = \frac{504}{140} = \frac{252}{70} = \frac{126}{35} = \frac{18}{5}.$

$$\frac{18}{5}) \; 18 \; (3\tfrac{3}{5}. \quad \text{Ans.}$$

$$\begin{array}{r} 15 \\ \hline 3 \end{array}$$

(56) (*a*) $.875 \div \frac{1}{2} = .875 \div .5 \left(\text{since } \frac{1}{2} = .5\right) = 1.75$. Ans.
Another way of solving this is to reduce .875 to its equivalent common fraction and then divide.

$.875 = \frac{7}{8}$, since $.875 = \frac{875}{1,000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; then, $\frac{7}{8} \div \frac{1}{2} = \frac{7}{8} \times \frac{2}{1} = \frac{7 \times 2}{8} = \frac{14}{8} = \frac{7}{4} = 1\frac{3}{4}$.

Since $\frac{3}{4} = $

$$
\begin{array}{r}
4\overline{)\,3.00\,}(.75 \\
2\,8 \\
\hline
2\,0 \\
2\,0 \\
\hline
\end{array}
$$

$1\frac{3}{4} = 1.75$, the same answer as above.

(*b*) $\frac{7}{8} \div .5 = \frac{7}{8} \div \frac{1}{2} \left(\text{since } .5 = \frac{1}{2}\right) = \frac{7}{8} \times \frac{2}{1} = \frac{7 \times 2}{8} = \frac{14}{8} = \frac{7}{4} = 1\frac{3}{4}$, or 1.75. Ans.

This can also be solved by reducing $\frac{7}{8}$ to its equivalent decimal and dividing by .5; $\frac{7}{8} = .875$; $.875 \div .5 = 1.75$. Since there are three decimal places in the dividend and one in the divisor, there are 3 — 1, or 2 decimal places in the quotient.

(*c*) $\dfrac{.375 \times \frac{1}{4}}{\frac{5}{16} - .125} = ?$ We will solve this problem by first reducing the decimals to their equivalent common fractions.

$.375 = \frac{375}{1,000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}$. $\frac{3}{8} \times \frac{1}{4} = \frac{3}{32}$, or the value of the numerator of the fraction.

$.125 = \frac{125}{1,000} = \frac{25}{200} = \frac{1}{8}$. Reducing $\frac{1}{8}$ to 16ths, we have $\frac{1 \times 2}{8 \times 2} = \frac{2}{16}$. Then, $\frac{5}{16} - \frac{2}{16} = \frac{3}{16}$, or the value of the

denominator of the fraction. The problem is now reduced to

$$\frac{\frac{3}{32}}{\frac{3}{16}} = ? \quad \frac{\frac{3}{32}}{\frac{3}{16}} = \frac{3}{32} \div \frac{3}{16} = \frac{3}{32} \times \frac{16}{3} = \frac{48}{96} = \frac{1}{2} \text{ or } .5. \quad \text{Ans.}$$

(57) (*a*) $(72 \times 48 \times 28 \times 5) \div (84 \times 15 \times 7 \times 6)$.

Placing the numerator over the denominator the problem becomes

$$\frac{72 \times 48 \times 28 \times 5}{84 \times 15 \times 7 \times 6} = ?$$

The 5 in the numerator and 15 in the denominator are both *divisible* by 5, since 5 divided by 5 equals 1, and 15 divided by 5 equals 3. *Cross off* the 5 and write the 1 *over* it; also *cross off* the 15 and write the 3 *under* it. Thus,

$$\frac{72 \times 48 \times 28 \times \overset{1}{\cancel{5}}}{84 \times \underset{3}{\cancel{15}} \times 7 \times 6} =$$

72 in the numerator and 84 in the denominator are *divisible* by 12, since 72 divided by 12 equals 6, and 84 divided by 12 equals 7. *Cross off* the 72 and write the 6 *over* it; also, *cross off* the 84 and write the 7 *under* it. Thus,

$$\frac{\overset{6}{\cancel{72}} \times 48 \times 28 \times \overset{1}{\cancel{5}}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{15}} \times 7 \times 6} =$$

Again, 28 in the numerator and 7 in the denominator are *divisible* by 7, since 28 divided by 7 equals 4, and 7 divided by 7 equals 1. *Cross off* the 28 and write the 4 *over* it; also, *cross off* the 7 and write the 1 *under* it. Thus,

$$\frac{\overset{6}{\cancel{72}} \times 48 \times \overset{4}{\cancel{28}} \times \overset{1}{\cancel{5}}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{15}} \times \underset{1}{\cancel{7}} \times 6} =$$

Again, 48 in the numerator and 6 in the denominator are *divisible* by 6, since 48 divided by 6 equals 8, and 6 divided by 6 equals 1. *Cross off* the 48 and write the 8 *over* it; also, *cross off* the 6 and write the 1 *under* it. Thus,

$$\frac{\overset{6}{\cancel{72}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \overset{1}{\cancel{5}}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{15}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{6}}} =$$

Again, 6 in the numerator and 3 in the denominator are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and write the 2 *over* it; also, *cross off* the 3 and write the 1 *under* it. Thus

$$\frac{\overset{\overset{2}{\cancel{6}}}{\cancel{72}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \overset{1}{\cancel{5}}}{\underset{7}{\cancel{84}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{15}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{6}}}.$$

Since there are *no two remaining numbers* (one in the numerator and one in the denominator) *divisible* by *any number* except 1, without a remainder, it is *impossible* to cancel further.

Multiply all the *uncanceled numbers* in the numerator together, and divide their *product* by the *product* of all the *uncanceled* numbers in the denominator. The *result* will be the *quotient*. The *product* of all the *uncanceled numbers* in the numerator equals $2 \times 8 \times 4 \times 1 = 64$; the product of all the *uncanceled* numbers in the denominator equals $7 \times 1 \times 1 \times 1 = 7$.

Hence, $$\frac{\overset{\overset{2}{\cancel{6}}}{\cancel{72}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \overset{1}{\cancel{5}}}{\underset{7}{\cancel{84}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{15}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{6}}} = \frac{2 \times 8 \times 4 \times 1}{7 \times 1 \times 1 \times 1} = \frac{64}{7} = 9\frac{1}{7}. \text{ Ans.}$$

(*b*) $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20)$.

Placing the numerator over the denominator, the problem becomes
$$\frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$$

The 50 in the numerator and 70 in the denominator are both *divisible* by 10, since 50 divided by 10 equals 5, and 70 divided by 10 equals 7. *Cross off* the 50 and write the 5 *over* it; also, *cross off* the 70 and write the 7 *under* it. Thus,

$$\frac{80 \times 60 \times \overset{5}{\cancel{50}} \times 16 \times 14}{\underset{7}{\cancel{70}} \times 50 \times 24 \times 20} =$$

Also, 80 in the numerator and 20 in the denominator are *divisible* by 20, since 80 divided by 20 equals 4, and 20 divided by 20 equals 1. *Cross off* the 80 and write the 4 *over* it; also, cross off the 20 and write the 1 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times 60 \times \overset{5}{\cancel{50}} \times 16 \times 14}{\underset{7}{\cancel{70}} \times 50 \times 24 \times \underset{1}{\cancel{20}}} =$$

Again, 16 in the numerator and 24 in the denominator are *divisible* by 8, since 16 divided by 8 equals 2, and 24 divided by 8 equals 3. *Cross off* the 16 and write the 2 *over* it; also, cross off the 24 and write the 3 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times 60 \times \overset{5}{\cancel{50}} \times \overset{2}{\cancel{16}} \times 14}{\underset{7}{\cancel{70}} \times 50 \times \underset{3}{\cancel{24}} \times \underset{1}{\cancel{20}}} =$$

Again, 60 in the numerator and 50 in the denominator are *divisible* by 10, since 60 divided by 10 equals 6, and 50 divided by 10 equals 5. *Cross off* the 60 and write the 6 *over* it; also, cross off the 50 and write the 5 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{6}{\cancel{60}} \times \overset{5}{\cancel{50}} \times \overset{2}{\cancel{16}} \times 14}{\underset{7}{\cancel{70}} \times \underset{5}{\cancel{50}} \times \underset{3}{\cancel{24}} \times \underset{1}{\cancel{20}}} =$$

The 14 in the numerator and 7 in the denominator are *divisible* by 7, since 14 divided by 7 equals 2, and 7 divided by 7 equals 1. *Cross off* the 14 and write the 2 *over* it; also, cross off the 7 and write the 1 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{6}{\cancel{60}} \times \overset{5}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{70}} \times \underset{5}{\cancel{50}} \times \underset{3}{\cancel{24}} \times \underset{1}{\cancel{20}}} =$$

The 5 in the numerator and 5 in the denominator are *divisible* by 5, since 5 divided by 5 equals 1. *Cross off* the

5 of the *dividend* and write the 1 *over* it; also, cross off the 5 of the *divisor* and write the 1 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{6}{\cancel{60}} \times \overset{\overset{1}{\cancel{5}}}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{70}} \times \underset{\underset{1}{\cancel{5}}}{\cancel{50}} \times \underset{\underset{1}{3}}{\cancel{24}} \times \underset{1}{\cancel{20}}} =$$

The 6 in the numerator and 3 in the denominator are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and place 2 *over* it; also, cross off the 3 and place 1 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{\overset{2}{\cancel{6}}}{\cancel{60}} \times \overset{\overset{1}{\cancel{5}}}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{70}} \times \underset{\underset{1}{\cancel{5}}}{\cancel{50}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{24}} \times \underset{1}{\cancel{20}}} =$$

Hence, $\dfrac{\overset{4}{\cancel{80}} \times \overset{\overset{2}{\cancel{6}}}{\cancel{60}} \times \overset{\overset{1}{\cancel{5}}}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{70}} \times \underset{\underset{1}{\cancel{5}}}{\cancel{50}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{24}} \times \underset{1}{\cancel{20}}} = \dfrac{4 \times 2 \times 1 \times 2 \times 2}{1 \times 1 \times 1 \times 1} = \dfrac{32}{1} = 32.$ Ans.

(58) (*a*) $\dfrac{7}{\dfrac{3}{16}} = 7 \div \dfrac{3}{16} = 7 \times \dfrac{16}{3} = \dfrac{7 \times 16}{3} = \dfrac{112}{3} = 37\dfrac{1}{3}.$ Ans.

The heavy line indicates that 7 is to be divided by $\dfrac{3}{16}$.

(*b*) $\dfrac{\dfrac{15}{32}}{\dfrac{5}{8}} = \dfrac{15}{32} \div \dfrac{5}{8} = \dfrac{15}{32} \times \dfrac{8}{5} = \dfrac{\overset{3}{\cancel{15}} \times \cancel{8}}{\underset{4}{\cancel{32}} \times \cancel{5}} = \dfrac{3}{4} = .75.$ Ans.

(*c*) $\dfrac{1.25 \times 20 \times 3}{\dfrac{87+88}{459+32}} = ?$ In this problem $1.25 \times 20 \times 3$ constitutes the numerator of the complex fraction.

$$\begin{array}{lr} & 1.25 \\ \times & 20 \\ \hline & 25.00 \\ \times & 3 \\ \hline & 75 \end{array}$$

Multiplying the factors of the numerator together, we find their product to be 75.

The fraction $\frac{87+88}{459+32}$ constitutes the denominator of the complex fraction. The value of the numerator of this fraction equals $87+88=175$.

The value of the denominator of this fraction is equal to $459+32=491$. The problem then becomes

$$\frac{75}{\frac{175}{491}} = \frac{\frac{75}{1}}{\frac{175}{491}} = \frac{75}{1} \div \frac{175}{491} = \frac{75}{1} \times \frac{491}{175} = \frac{\overset{3}{\cancel{75}} \times 491}{\underset{7}{\cancel{175}}} = \frac{1,473}{7} = 210\tfrac{3}{7}.$$

Ans.

(59) The pitch of the rivets is the distance between the centers of the rivets. Hence, since the distance around the cylindrical boiler is 166.85 in., and there are 72 rivets in one of the seams, the pitch of the rivets equals $166.85 \div 72 = 2.317+$ in. Ans.

```
72 ) 166.850 ( 2.317+
     144
     ---
      228
      216
      ---
       125
        72
       ---
        530
        504
        ---
         26
```

(60) If a keg containing 133 boiler rivets weighs 100 pounds, then each rivet must weigh as much as 133 is contained times in 100, or .75 of a pound.

```
133 ) 100.00 ( .75+   Ans.
       931
       ---
        690
        665
        ---
         25
```

Since there are 2 decimal places in the dividend and 0 decimal places in the divisor, we must point off $2-0=2$ decimal places in the quotient, or answer.

(61) If the distance around a fly-wheel is 56.5488 ft., which is 3.1416 times its diameter, then its diameter must equal $56.5488 \div 3.1416 = 18$ ft.

$$\begin{array}{r|l} 3.1416\,)\,56.5488\,(\,18\text{ ft.} & \\ 31416 & \\ \hline 251328 & \\ 251328 & \\ \hline 0 & \end{array}$$

If the diameter of the fly-wheel is 18 ft., then it is evident that the diameter of a wheel one-half as large must be $\frac{1}{2}$ of 18 ft., or 9 ft.

(62) Since 1 load consists of 1,600 bricks, 8 loads consist of $8 \times 1,600 = 12,800$ bricks. Since 1 team can draw 12,800 bricks in 1 day, 6 teams can draw 6 times as many, or $6 \times 12,800 = 76,800$ bricks in 1 day. Since 6 teams can draw 76,800 bricks in 1 day, to draw 980,000 bricks, it will take them as many days as 76,800 is contained times in 980,000, or 12.76 + days. Ans.

$$\begin{array}{r} 76800\,)\,980000.00\,(\,12.76+ \\ 76800 \\ \hline 212000 \\ 153600 \\ \hline 584000 \\ 537600 \\ \hline 464000 \\ 460800 \\ \hline 3200 \end{array}$$

Since there are 2 decimal places in the dividend and 0 decimal places in the divisor, we must point off $2 - 0 = 2$ decimal places in the quotient.

(63) 28 acres of land at $133 an acre would cost $28 \times \$133 = \$3,724$.

$$\begin{array}{r} 28 \\ \hline 1064 \\ 266 \\ \hline \$3724 \end{array}$$

If a mechanic earns $1,500 a year, and his expenses are $968 per year, then he would save $1 5 0 0 — $968, or $532 per year.

$$\begin{array}{r} \$1500 \\ 968 \\ \hline \$532 \end{array}$$

If he saves $532 in one year, to save $3,724 it would take as many years as $532 is contained times in $3,724, or 7 years.

$$\begin{array}{r} 532\,)\,3724\,(\,7 \text{ years. Ans.} \\ 3724 \\ \hline 0 \end{array}$$

(64) $\frac{7}{8}$ = value of the fraction, and 28 the numerator. We find that 4 multiplied by $7 = 28$, so multiplying 8, the denominator of the fraction, by 4, we have 32 for the required denominator, and $\frac{28}{32} = \frac{7}{8}$. Hence, 32 is the required denominator.

(65) 1 plus $.001 = 1.001$. $.01$ plus $.000001 = .010001$. And $1.001 - .010001 =$

$$\begin{array}{r} 1.001 \\ .010001 \\ \hline .990999 \end{array} \text{ Ans.}$$

(66) If the freight train ran 365 miles in one week, and 3 times as far, lacking 246 miles the next week, then it ran (3×365 miles) — 246 miles, or 849 miles the second week. Thus,

$$\begin{array}{r} 365 \\ 3 \\ \hline 1095 \\ -\ 246 \\ \hline 849 \end{array} \text{ miles. Ans.}$$

(67) The distance from Philadelphia to Pittsburg is 354 miles. Since there are 5,280 feet in 1 mile, in 354 miles there are $354 \times 5,280$ feet, or 1,869,120 feet. If the driving wheel of the locomotive is 16 feet in circumference, in going

from Philadelphia to Pittsburg, a distance of 1,869,120 feet, it will make 1,869,120 ÷ 16, or 116,820 revolutions.

```
16 ) 1869120 ( 116820 rev   Ans.
     16
     ---
      26
      16
      ---
      109
       96
      ---
       131
       128
       ---
         32
         32
         --
          0
```

(68) $.875 = \frac{875}{1{,}000} = \frac{175}{200} = \frac{7}{8}$ of a foot.

1 foot = 12 inches.

$$\frac{7}{8} \text{ of 1 foot} = \frac{7}{\underset{2}{\cancel{8}}} \times \frac{\overset{3}{\cancel{12}}}{1} = \frac{21}{2} = 10\frac{1}{2} \text{ inches.} \quad \textbf{Ans.}$$

(69) 12 inches = 1 foot.

$$\frac{3}{16} \text{ of an inch} = \frac{3}{16} \div 12 = \frac{\overset{1}{\cancel{3}}}{16} \times \frac{1}{\underset{4}{\cancel{12}}} = \frac{1}{64} \text{ of a foot.}$$

```
 1
 --
64) 1.000000 ( .015625 Ans.
     64
     ---
     360
     320
     ---
      400
      384
      ---
       160
       128
       ---
        320
        320
        ---
          0
```

Point off 6 decimal places in the quotient, since we annexed six ciphers to the dividend, the divisor containing no decimal places; hence, 6 − 0 = 6 places to be pointed off.

(70) 6 inches $= \frac{6}{12}$ of a foot, since 12 inches = 1 foot.

12 feet 6 inches $= 12\frac{6}{12} = 12\frac{1}{2}$ feet $= \frac{(12 \times 2) + 1}{2} = \frac{25}{2}$ feet.

If there are 7 pieces of pipe and each piece is $12\frac{1}{2}$ feet long, then the whole length of the pipe would be $\frac{25}{2} \times 7 = \frac{175}{2} = 87.5$ feet.

12 inches = 1 foot. And $\frac{3}{4}$ inch $= \frac{3}{4} \div 12 = \frac{3}{4} \div \frac{12}{1} = \frac{\cancel{3}}{4} \times \frac{1}{\cancel{12}_{4}} = \frac{1}{16}$ of a foot to be allowed at each joint for screwing.

$\frac{1}{16}$ reduced to its equivalent decimal = .0625 of a foot.

The first length screws into the tank $\frac{1}{16}$ or .0625 of a foot, thereby shortening the length of the pipe .0625 of a foot. The length of the pipe now equals 87.5 feet − .0625 foot = 87.4375 feet.

$$\begin{array}{rl} 87.5 & \text{feet} \\ -\quad .0625 & \text{feet} \\ \hline 87.4375 & \text{feet} \end{array}$$

The $\frac{3}{4}$ of an inch, or $\frac{1}{16}$ of a foot, allowed at each of the other 6 joints must be added to the length of the pipe since the different lengths are connected by unions which prevent the ends of the pipe from coming together, and, in this case, keep them $\frac{3}{4}''$ apart. Hence, we have $6 \times \frac{1}{16} = \frac{6}{16}$ or .375 of a foot for the 6 joints.

These 6 joints lengthen the pipe .375 of a foot; consequently, the water will be discharged at a distance of 87.4375 feet + .375 foot, or 87.8125 feet from the tank.

$$\begin{array}{rl} 87.4375 & \text{feet} \\ +\quad .375 & \text{feet} \\ \hline 87.8125 & \text{feet} \quad \text{Ans.} \end{array}$$

(71) 72.6 feet of fencing at \$.50 a foot would cost $72.6 \times .50$, or \$36.30.

$$\begin{array}{r} .50 \\ \hline \$36.300 \end{array}$$

If, by selling a carload of coal at a profit of \$1.65 per ton, I make \$36.30, then there must be as many tons of coal in the car as 1.65 is contained times in 36.30, or 22 tons.

$$\begin{array}{r} 1.65)\,36.30\,(22 \text{ tons. Ans.} \\ 330\phantom{(22 \text{ tons. Ans.}} \\ \hline 330\phantom{(22 \text{ tons. Ans.}} \\ 330\phantom{(22 \text{ tons. Ans.}} \\ \hline 0\phantom{(22 \text{ tons. Ans.}} \end{array}$$

(72) 14 ft. 5 in. $= 14\frac{5}{12}$ ft., since 12 inches equal 1 foot. Of the six pieces of connecting pipe between an engine and boiler, there are three pieces each measuring 14 ft. 5 in., or $14\frac{5}{12}$ ft.; hence, the total length of the three pieces equals $3 \times 14\frac{5}{12} = 3 \times \frac{173}{12} = \frac{519}{12} = 43\frac{1}{4}$ ft.

Since each of the other two pieces is 12 ft. 6 in., or $12\frac{1}{2}$ ft. in length, the total length of these two pieces equals $2 \times 12\frac{1}{2}$ ft. $= 25$ ft. The remaining piece of pipe is 8 ft. 10 in., or $8\frac{5}{6}$ ft. in length, since 10 in. $= \frac{10}{12}$, or $\frac{5}{6}$ of a ft. (there being 12 inches in 1 foot). Adding these different lengths will give the total length of the six pieces of pipe. However, before adding we will reduce the fractions of the two mixed numbers $43\frac{1}{4}$ and $8\frac{5}{6}$ to fractions having a common denominator. Doing this, we have $\frac{1}{4} = \frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$; $\frac{5}{6} = \frac{5}{6} \times \frac{2}{2} = \frac{10}{12}$. $\frac{3}{12} + \frac{10}{12} = \frac{13}{12} = 1\frac{1}{12}$ ft. The sum of the whole

numbers equals 43 ft. + 25 ft. + 8 ft. = 76 ft., which added to $1\frac{1}{12}$ ft. gives $77\frac{1}{12}$ ft. as the total length of the connecting pipe. Since 1 ft. of pipe weighs $10\frac{1}{2}$ pounds, then it is clearly seen that this pipe would weigh $77\frac{1}{12} \times 10\frac{1}{2} = \frac{925}{\cancel{12}_{4}} \times \frac{\cancel{21}^{7}}{2} = \frac{6,475}{8} = 809\frac{3}{8}$, or 809.375 lb. Ans.

$$\begin{array}{r} 77 \\ \times\ 12 \\ \hline 154 \\ 77 \\ \hline 924 \\ +\ \ 1 \\ \hline 925 \end{array} \qquad \frac{3}{8} = .375, \text{ since } 8\,\underline{)\,3.000} \atop .375$$

(73) Since these four bolts measure $2\frac{1}{2}$, $6\frac{7}{8}$, $3\frac{1}{16}$, and 4 inches, respectively, together they will measure $16\frac{7}{16}$ inches, since $2\frac{1}{2} + 6\frac{7}{8} + 3\frac{1}{16} + 4 = 16\frac{7}{16}$. Reducing the fractions of the mixed numbers to a common denominator, we have $\frac{1}{2} = \frac{1}{2} \times \frac{8}{8} = \frac{8}{16}$; $\frac{7}{8} = \frac{7}{8} \times \frac{2}{2} = \frac{14}{16}$; $\frac{8}{16} + \frac{14}{16} + \frac{1}{16} = \frac{8+14+1}{16} = \frac{23}{16} = 1\frac{7}{16}$. $2 + 6 + 3 + 4 + 1\frac{7}{16} = 16\frac{7}{16}$. If $\frac{7}{16}$ of an inch is allowed for cutting and finishing each bolt, then the allowance for the 4 bolts would equal $4 \times \frac{7}{16} = \frac{28}{16} = 1\frac{12}{16}$ inches, which added to $16\frac{7}{16}$ inches equals $18\frac{3}{16}$ inches, the length of the piece of iron required. Ans.

$$\begin{array}{r} 16\frac{7}{16} \\ 1\frac{12}{16} \\ \hline 17\frac{19}{16} \end{array}, \text{ or } 18\frac{3}{16}, \text{ since } \frac{19}{16} = 1\frac{3}{16}.$$

(74) A double belt of a certain width can transmit $64\frac{1}{2}$, or 64.5 horsepower. If this is $\frac{10}{7}$ of the power transmitted by one single belt, then $\frac{1}{7}$ of the power transmitted would equal $\frac{1}{10}$ of 64.5, or 6.45 horsepower, and $\frac{7}{7}$ would equal 7×6.45 horsepower, or 45.15 horsepower. Since 1 single belt of a certain width can transmit 45.15 horsepower, it is clearly evident that 2 single belts, when running under the same conditions, would transmit 2×45.15, or 90.3 horsepower. Ans.

(75) Before solving, we will reduce the several lengths to the same denomination, in this case feet. Since 12 inches = 1 foot,

18 ft. 6 in. $= 18\frac{6}{12}$, or $18\frac{1}{2}$ ft.;

16 ft. $9\frac{1}{2}$ in. $= 16\frac{9\frac{1}{2}}{12} = 16\frac{\frac{19}{2}}{12} = 16\frac{19}{24}$;

$$\left(\frac{19}{2} \div 12 = \frac{19}{2} \div \frac{12}{1} = \frac{19}{2} \times \frac{1}{12} = \frac{19}{24}\right);$$

22 ft. 2 in. $= 22\frac{2}{12}$, or $22\frac{1}{6}$ ft.;

20 ft. $8\frac{1}{2}$ in. $= 20\frac{8\frac{1}{2}}{12} = 20\frac{\frac{17}{2}}{12} = 20\frac{17}{24}$.

$$\left(\frac{17}{2} \div 12 = \frac{17}{2} \div \frac{12}{1} = \frac{17}{2} \times \frac{1}{12} = \frac{17}{24}\right).$$

Adding the different lengths, we have

$$18\frac{1}{2} \text{ ft.} + 16\frac{19}{24} \text{ ft.} + 22\frac{1}{6} \text{ ft.} + 20\frac{17}{24} \text{ ft.} = 78\frac{1}{6} \text{ ft.}$$

Reducing $\frac{1}{2}$, $\frac{19}{24}$, $\frac{1}{6}$, and $\frac{17}{24}$ to fractions having a common denominator, as in problem 42, we have $\frac{1}{2} = \frac{12}{24}$; $\frac{19}{24} = \frac{19}{24}$; $\frac{1}{6} = \frac{4}{24}$, and $\frac{17}{24} = \frac{17}{24}$. Adding, we have $\frac{12 + 19 + 4 + 17}{24} =$

$\frac{52}{24} = 2\frac{4}{24}$, or $2\frac{1}{6}$, which added to the sum of the whole numbers, or 76, equals $78\frac{1}{6}$ ft. Ans.

$$\begin{array}{rl} 18\frac{1}{2} & \text{ft.} \\ 16\frac{11}{24} & \text{ft.} \\ 22\frac{1}{6} & \text{ft.} \\ 20\frac{17}{24} & \text{ft.} \\ \hline 76 & \text{ft.} \\ 2\frac{1}{6} & \text{ft.} \\ \hline 78\frac{1}{6} & \text{ft.} \end{array}$$

(76) The total horsepower developed equals 48.63 + 45.7 + 46.32 + 47.9 + 48.74 + 48.38 + 48.59 = 334.26.

Since the horsepower developed equals 334.26, then the average horsepower developed must equal 334.26 ÷ 7, or 47.75 + H. P.

$$\begin{array}{r} 48.63 \\ 45.7 \\ 46.32 \\ 47.9 \\ 48.74 \\ 48.38 \\ 48.59 \\ \hline 334.26 \end{array}$$

ARITHMETIC.

(QUESTIONS 77-141.)

(77) A certain per cent. of a number means so many hundredths of that number.

25% of 8,428 lb. means 25 hundredths of 8,428 lb. $\frac{25}{100} =$ .25. Hence, 8,428 lb. $\times$ .25 = 2,107 lb. Ans.

(78) 1% means one-hundredth $\left(\frac{1}{100}\right)$, which is expressed decimally as .01. Then, \$100 $\times$ .01 = \$1.

$$\begin{array}{r} \$1\,0\,0 \\ .0\,1 \\ \hline \$1.0\,0 \end{array} \text{ Ans.}$$

(79) $\frac{1}{2}$% means one-half of 1 per cent. Since 1% is .01, $\frac{1}{2}$% is .005, for $2\,\underline{).0\,1\,0}$ $\;.0\,0\,5$. And \$35,000 $\times$ .005 = \$175. Ans.

$$\begin{array}{r} \$3\,5\,0\,0\,0 \\ .0\,0\,5 \\ \hline \$1\,7\,5.0\,0\,0 \end{array}$$

(80) If 2 is a certain per cent. of 50, then 50 multiplied by a certain rate gives a product of 2, and that rate is equal to 2 divided by 50. Dividing 2 by 50, the quotient is .04, which means that 2 is 4% of 50, or, since percentage = base $\times$ rate,

$$\begin{array}{l} 5\,0\,)\,2.0\,0\,(\,.0\,4 \text{ Ans.} \\ \quad\;\; 2\,0\,0 \\ \quad\;\; \overline{} \end{array}$$

rate = percentage $\div$ base
= $2 \div 50 = .04$, or 4%. Ans.

(81) Since percentage = base × rate, rate = percentage ÷ base.*

As percentage = 10 and base = 10, we have rate = 10 ÷ 10 = 1.

But $1 = \frac{100}{100}$ and $\frac{100}{100} = 100\%$; hence, the rate (1) means that 10 is 100% of 10.

(82) (*a*) Rate = percentage ÷ base.

As percentage = \$176.54 and base = \$2,522, we have rate = 176.54 ÷ 2,522 = .07 = 7%. Ans.

```
2522 ) 176.54 ( .07
       17654
       -----
```

(*b*) Base = percentage ÷ rate.

As percentage = 16.96 and rate = 8% = .08, we have base = 16.96 ÷ .08 = 212. Ans.

```
.08 ) 16.96 ( 212
      16
      --
        9
        8
        --
        16
        16
        --
```

(*c*) Amount is the sum of the base and percentage; hence, the percentage = amount minus the base.

Amount = 216.7025 and base = 213.5; hence, percentage = 216.7025 − 213.5 = 3.2025.

Rate = percentage ÷ base.

As percentage = 3.2025 and base = 213.5, we have rate = 3 2025 ÷ 213.5 = .015. Ans.

```
213.5 ) 3.2025 ( .015 = 1½%
        2135
        -----
        10675
        10675
        -----
```

***Remember that an expression of this form means that the first term is to be divided by the second term. Thus, as above, it means percentage divided by base.*

(*d*) The difference is the remainder found by subtracting the percentage from the base; hence, base — the difference = the percentage. Base = 207 and difference = 201.825; hence, percentage = 207 — 201.825 = 5.175.

Rate = percentage ÷ base.

As percentage = 5.175 and base = 207, we have rate = 5.175 ÷ 207 = $.025 = .02\frac{1}{2} = 2\frac{1}{2}\%$. Ans.

```
207 ) 5.175 ( .025
      414
      ----
      1035
      1035
      ----
```

(83) Since 5,500 lb. represent an increase of 15% over the consumption when the condenser is used, 5,500 lb. must be the amount, 15% the rate, and the number of pounds consumed when the condenser is running (to be found) the base.

Base = amount ÷ (1 + rate) = 5,500 ÷ (1 + .15)
= 5,500 ÷ 1.15 = 4,782.61 lb., nearly. Ans.

```
1.15 ) 5500.0000 ( 4782.61
       460
       ---
        900
        805
        ---
         950
         920
         ---
          300
          230
          ---
           700
           690
           ---
            100
            115
            ---
```

Or, this problem could also have been solved as follows:

100% = the number of pounds consumed when the condenser is running. If there is a gain of 15%, then 100% + 15%, or 115% = 5,500 lb., the amount used when the condenser is not running. If 115% = 5,500 lb., $1\% = \frac{1}{115}$ of 5,500 = 47.8261 lb., and 100% = 100 × 47.8261 = 4,782.61 lb. Ans.

(84)

$$
\begin{aligned}
24\ \% \text{ of } \$950 &= 950 \times .24 &&= \$228\\
12\tfrac{1}{2}\% \text{ of } \$950 &= 950 \times .125 &&= 118.75\\
17\ \% \text{ of } \$950 &= 950 \times .17 &&= 161.50\\
\hline
53\tfrac{1}{2}\% \text{ of } \$950 & &&= \$508.25
\end{aligned}
$$

The total amount of his yearly expenses, then, is \$508.25; hence, his savings are \$950 − \$508.25 = \$441.75. Ans.

Or, as above, $24\% + 12\frac{1}{2}\% + 17\% = 53\frac{1}{2}\%$, the total percentage of expenditures; hence,

$\$950 \times .535 = \$508.25$, and

$\$950 - \$508.25 = \$441.75 =$ his yearly savings. Ans.

(85) The percentage is 961.38 and the rate is $37\frac{1}{2}\%$.

Base = percentage ÷ rate

= 961.38 ÷ .375 = 2,563.68, the number. Ans.

```
.375 ) 961.38000 ( 2563.68
       750
       ----
       2113
       1875
       -----
        2388
        2250
        -----
         1380
         1125
         -----
          2550
          2250
          -----
           3000
           3000
           ----
```

Another method of solving is the following:

If $37\frac{1}{2}\%$ of a number is 961.38, then $.37\frac{1}{2}$ times the number = 961.38 and the number = $961.38 \div .37\frac{1}{2}$, which, as above = 2,563.68. Ans.

(86) Here \$1,125 is 30% of some number; hence, \$1,125 = the percentage, 30% = the rate, and the required number is the base.

Base = percentage ÷ rate.

= \$1,125 ÷ .30 = \$3,750.

Since \$3,750 is $\frac{3}{4}$ of the property, one of the fourths is $\frac{1}{3}$

of \$3,750 = \$1,250, and $\frac{4}{4}$, or the entire property, is 4 × \$1,250 = \$5,000. Ans.

(87) Here, \$4,810 is the difference and 35% the rate.
Base = difference ÷ (1 − rate)
= \$4,810 ÷ (1 − .35) = \$4,810 ÷ .65 = \$7,400. Ans.

```
.65 ) 4810.00 ( 7400
      455
      ---
       260                  1.00
       260                   .35
       ---                   ---
                             .65
```

The solution can also be effected as follows: 100% = the sum diminished by 35%, then (1 − .35) = .65, which is \$4,810.

If 65% = \$4,810, 1% = $\frac{1}{65}$ of 4,810 = \$74, and 100% = 100 × \$74 = \$7,400. Ans.

(88) The volume of the clearance in a steam-engine cylinder = 18.3 cu. in., and the volume of the cylinder, neglecting the clearance, = 254.5 cu. in. We wish to know what percentage of the cylinder volume is the clearance, or what per cent. of 254.5 is 18.3.

254.5 is the base.

18.3 is the percentage.

The rate, which we wish to find, equals the percentage ÷ base = 18.3 ÷ 254.5 = 7.2% nearly.

```
254.5 ) 18.3000 (.072, or 7.2%, nearly.
        17815
        -----
          4850
          5090
          ----
```

(89) 16.5 miles = $12\frac{1}{2}$% of the entire length of the road

We wish to find the *entire* length.

16.5 miles is the percentage and $12\frac{1}{2}$% is the rate, and the entire length will be the base.

$$\text{Base} = \text{percentage} \div \text{rate}$$
$$= 16.5 \div .12\tfrac{1}{2}.$$

```
.125)16.500(132 miles.   Ans.
      125
      ---
       400
       375
       ---
        250
        250
        ---
```

(90) 298 revolutions per minute with the load = base. $1\frac{1}{2}\%$ = rate, and the amount (to be found) will equal the speed of the engine when running unloaded.

Amount = base × (1 + rate)

$= 298 \times (1 + .015) = 302.47$ rev. per min. Ans.

```
      298
  ×  1.015
  -------
     1490
     298
    000
   298
  -------
   302.470
```

(91) 4 yd. 2 ft. 10 in. to inches.

```
 ×   3
    --
    12
 +   2
    --
    14 feet
 ×  12
    --
    28
   14
   ---
   168
 +  10
   ---
   178 inches.   Ans.
```

Since there are 3 feet in one yard, in 4 yards there are 4 × 3 feet, or 12 feet. 12 feet plus 2 feet = 14 feet.

There are 12 inches in one foot; therefore, in 14 feet there are 12 × 14 or 168 inches. 168 inches plus 10 inches = 178 inches. Ans.

(92)

$$\begin{array}{r l} 12\,)\,\underline{3722} & \text{inches.} \\ 3\,)\,\underline{310} & +\,2 \text{ inches.} \\ 103 & +\,1 \text{ foot.} \end{array}$$

Ans. = 103 yd. 1 ft. 2 in.

Explanation.—There are 12 inches in one foot; hence, in 3,722 inches there are as many feet as 12 is contained times in 3,722, or 310 feet and 2 inches remaining. Write 2 inches as a remainder. There are 3 feet in one yard; hence, in 310 yards there are as many feet as 3 is contained times in 310, or 103 yards and 1 foot remaining. Hence, in 3,722 inches there are 103 yd. 1 ft. 2 in.

(93)

$$\begin{array}{r l} & 5 \text{ weeks } 3.5 \text{ days.} \\ \times & \underline{7} \\ & 35 \text{ days in 5 weeks.} \\ + & \underline{3.5} \\ & 38.5 \text{ days.} \end{array}$$

Then, we find how many seconds there are in 38.5 days.

$$\begin{array}{r l} 38.5 & \text{days.} \\ \times\ \ \underline{24} & \text{hours in one day.} \\ 1540 & \\ \underline{770\ \ } & \\ 924.0 & \text{hours in 38.5 days.} \\ \times\ \ \underline{60} & \text{minutes in one hour.} \\ 55440 & \text{minutes in 38.5 days.} \\ \times\ \ \underline{60} & \text{seconds in one minute.} \\ 3326400 & \text{seconds in 38.5 days. Ans.} \end{array}$$

(94)

$$\begin{array}{r l} 1728\,)\,\underline{764325} & \text{cu. in.} \\ 27\,)\,\underline{442} & +\,549 \text{ cu. in.} \\ 16 & \text{cu. yd.} +\,10 \text{ cu. ft.} \end{array}$$

Ans. = 16 cu. yd. 10 cu. ft. 549 cu. in.

Explanation.—There are 1,728 cubic inches in one cubic foot; hence, in 764,325 cu. in. there are as many cubic feet as 1,728 is contained times in 764,325, or 442 cubic feet, and 549 cubic inches remaining. Write the 549 cubic inches as a remainder. There are 27 cubic feet in one cubic yard; hence, in 442 cubic feet there are as many cubic yards as 27

is contained times in 442 cubic feet, or 16 cubic yards and 10 cubic feet remaining. Then, in 764,325 cubic inches there are 16 cu. yd. 10 cu. ft. 549 cu. in. Ans.

(95) There are $31\frac{1}{2}$ (31.5, since $\frac{1}{2} = .5$) gallons in 1 barrel; hence, in 4 barrels there are 4 times 31.5, or 126 gallons. 126 gallons plus 10 gallons = 136 gallons.

$$
\begin{array}{lrll}
 & \text{bbl.} & \text{gal.} & \text{qt.} \\
 & 4 & 10 & 3 \\
\times & 31.5 & & \\
\hline
 & 126.0 & & \\
+ & 10 & & \\
\hline
 & 136 & \text{gallons.} & \\
\times & 4 & & \\
\hline
 & 544 & & \\
+ & 3 & & \\
\hline
4) & 547 & \text{quarts.} & \\
\hline
 & 136\frac{3}{4} & \text{gallons.} & \\
 & & \text{Ans.} &
\end{array}
$$

There are 4 quarts in 1 gallon; hence, in 136 gallons there are 136 times 4 = 544 quarts.

544 quarts plus 3 quarts = 547 quarts. Since the tank under consideration contains 547 quarts, and since 4 quarts = 1 gallon, the tank would contain as many gallons of water as 4 is contained times in 547, or $136\frac{3}{4}$ gallons. Ans.

(96)

$$
\begin{array}{lrrr}
 & \text{T.} & \text{cwt.} & \text{lb.} \\
 & 16 & 8 & 75 \\
\times & 20 & & \\
\hline
 & 320 & & \\
+ & 8 & & \\
\hline
 & 328 & \text{cwt.} & \\
\times & 100 & & \\
\hline
 & 32800 & & \\
+ & 75 & & \\
\hline
 & 32875 & \text{lb.} &
\end{array}
$$

Since in one ton there are 20 cwt., in 16 tons there are $16 \times 20 = 320$ cwt. 320 cwt. + 8 cwt. = 328 cwt. There are 100 lb. in 1 cwt.; hence, in 328 cwt. there are $328 \times 100 = 32,800$ lb. 32,800 lb. + 75 lb. = 32,875 lb. Ans.

(97)

$$
\begin{array}{r}
100\overline{)25396}\text{ lb.} \\
20\overline{)253}\text{ cwt.} + 96\text{ lb.} \\
\overline{12}\text{ T.} + 13\text{ cwt.}
\end{array}
$$

There are 100 lb. in 1 cwt.; hence, in 25,396 lb. there are as many cwt. as 100 is contained times in 25,396, or 253 cwt. and 96 lb. remaining.

There are 20 cwt. in 1 ton, and in 253 cwt. there are as many tons as 20 is contained times in 253, or 12 tons and

13 cwt. remaining. Hence, 25,396 lb. = 12 T. 13 cwt. 96 lb. Ans.

(98) There are 2 pt. in 1 qt.; hence, in 25,396 pt. there are 12,698 qt. There are 4 qt. in one gal.; hence, in 12,698 qt. there are as many gal. as 4 is contained times in 12,698, or 3,174 gal. with 2 qt. remaining.

$$\begin{array}{r|l} 2 & \underline{25396} \text{ pt.} \\ 4 & \underline{12698} \text{ qt.} \\ 31.5 & \underline{3174} \text{ gal.} + 2 \text{ qt.} \\ & 100 \text{ bbl.} + 24 \text{ gal.} \end{array}$$

Since $31\frac{1}{2}$, or 31.5 gal. make 1 bbl., in 3,174 gal. there would be as many bbl. as 31.5 is contained times in 3,174, or 100 bbl. and 24 gal. remaining.

$$\begin{array}{r} 31.5\,)\,3174.0 \text{ gal. } (\,100 \text{ bbl.} \\ \underline{315} \\ 24.0 \end{array}$$

Hence, 25,396 pt. = 100 bbl. 24 gal. 2 qt. Ans.

(99) Arrange the different terms in columns, taking care to have like denominations in the same column. We begin to add at the right-hand column. 7 + 9 + 3 = 19 in.; since 12 in. = 1 ft., 19 in. = 1 ft. and 7 in. Place the 7 in. in the inches column, and reserve the 1 ft. to add to the sum of the feet. 2 + 1 + 2 + 1 (reserved) = 6 ft. Since 3 ft. = 1 yd., 6 ft. = 2 yd. and 0 ft. remaining. Place the 0 in the feet column and reserve the 2 yd. to add to the sum of the yards. 4 + 2 + 2 (reserved) = 8 yd., which we place in yards column. Ans. = 8 yd. 7 in.

yd.	ft.	in.	
2	2	3	
4	1	9	
	2	7	
8	0	7	Ans.

(100) Since "pints" is the lowest denomination, we will find their sum first. 5 + 1 + 1 = 7 pt. Since there are 2 pt. in 1 qt., in 7 pt. there are as many quarts as 2 is contained in 7, or 3 qt. and 1 pt. remaining. Place 1 pt. under pints column and add

gal.	qt.	pt.	
3	3	1	
6	0	1	
4	8	5	
16	2	1	Ans.

3 qt. to the sum of the quarts. $8 + 3 + 3$ (reserved) $= 14$ qt. Since there are 4 qt. in 1 gal., in 14 qt. there are as many gallons as 4 is contained in 14, or 3 gal., and 2 qt. remaining. Place 2 qt. under quarts column and reserve the 3 gal. to add to the sum of the gallons. $4 + 6 + 3 + 3$ (reserved) $=$ 16 gal. Hence, the answer is 16 gal. 2 qt. 1 pt.

(101) We will first reduce 115 ft. to yards. Since 3 ft. $=$ 1 yd., 115 ft. $=$ 38 yd. and 1 ft. Place the subtrahend under the minuend, so that like denominations are under each other. Since the inch is the lowest denomination, we subtract the inches first.

yd.	ft.	in.	
52	2	9	
38	1	0	
14	1	9	Ans.

0 in. subtracted from 9 in. $=$ 9 in., which we write in inches place in the remainder. 1 ft. subtracted from 2 ft. $=$ 1 ft., which we write in the remainder. 38 yd. subtracted from 52 yd. $=$ 14 yd., which we write in yards place in the remainder. Hence, the remainder, or answer $=$ 14 yd. 1 ft. 9 in.

(102) Since 10 gal. 2 qt. 1 pt. of machine oil is sold at one time, and 16 gal. 3 qt. at another time, together there was sold 27 gal. 1 qt. 1 pt. $0 + 1 = 1$ pt. We can not reduce 1 pt. to any higher denomination, so place it under pints column. 3 qt. $+$ 2 qt. $=$ 5 qt. Since 4 qt. $=$ 1 gal., 5 qt. $=$ 1 gal., and 1 qt. remaining. Place 1 qt. under quarts column and reserve the 1 gal. to add to the gallons. 16 gal. $+$ 10 gal. $+$ 1 gal. (reserved) $=$ 27 gal. Since the barrel contained $31\frac{1}{2}$, or 31.5 gal., and 27 gal. 1 qt. 1 pt. were sold, there remained the difference, or 4 gal. 1 pt. 31.5 gal. $=$ 31 gal. 2 qt., since $.5 = \frac{1}{2}$, and $\frac{1}{2}$ of 1 gal. $=$ $\frac{1}{2}$ of 4 qt. $=$ 2 qt. 1 pt. can not be taken from 0 pt., so we take 1 qt. from the 2 qt. The 1 qt. taken $=$ 2 pt. 1 pt. from 2

gal.	qt.	pt.
10	2	1
16	3	0
27	1	1

gal.	qt.	pt.	
31	2	0	
27	1	1	
4	0	1	Ans.

pt. = 1 pt. Place 1 pt. under pints column. Since we took 1 qt. from the quarts column, there remains 2 − 1, or 1 qt. 1 qt. from 1 qt. leaves 0 qt. Place 0 qt. under the quarts column. 27 gal. from 31 gal. leaves 4 gal. Place 4 gal. under the gallons column. We, therefore, find that 4 gal. 1 pt. of machine oil remained in the barrel.

(103) In multiplication of denominate numbers, we place the multiplier under the lowest denomination of the multiplicand, as

$$\begin{array}{r} 17 \text{ ft. } 3 \text{ in.} \\ 51 \phantom{\text{ in.}} \\ \hline 879 \text{ ft. } 9 \text{ in.} \end{array}$$

and begin at the right to multiply. $51 \times 3 = 153$ in. Since there are 12 in. in 1 ft., in 153 in. there are as many feet as 12 is contained times in 153, or 12 ft. and 9 in. remaining. Place the 9 in. under the inches and reserve the 12 ft. 51×17 ft. $= 867$ ft.; 867 ft. + 12 ft. (reserved) = 879 ft. 879 ft. can be reduced to higher denominations by dividing by 3 ft. to find the number of yards, and by $5\frac{1}{2}$ to find the number of rods.

$$\begin{array}{r} 3\,\underline{)879} \text{ ft. } 9 \text{ in.} \\ 5.5\,\underline{)293} \text{ yd.} \phantom{\text{ 9 in.}} \\ 53 \text{ rd. } 1\tfrac{1}{2} \text{ yd.} \end{array}$$

Then, Ans. = 53 rd. $1\frac{1}{2}$ yd. 0 ft. 9 in., or 53 rd. 1 yd. 2 ft. 3 in.

(104) Since 2 pt. = 1 qt., 3 qt. = 3×2, or 6 pt. 6 pt. + 1 pt. = 7 pt. $4.7 \times 7 = 32.9$ pt. Ans.

$$\begin{array}{rl} \text{qt.} & \text{pt.} \\ 3 & 1 \\ \times\ 2 & \\ \hline 6 & \\ +\ 1 & \\ \hline 7 \text{ pt.} & \end{array} \qquad\qquad \begin{array}{r} 7 \text{ pt.} \\ 4.7 \phantom{\text{ pt.}} \\ \hline 49 \phantom{\text{ pt.}} \\ 28 \phantom{\text{ pt.}} \\ \hline 32.9 \text{ pt.} \end{array}$$

(105) We must first reduce 23 miles to feet before we can divide by 30 feet.

$$1 \text{ mi.} = 320 \text{ rd.}$$
$$23 \text{ mi.} = 23 \times 320 = 7{,}360 \text{ rd.}$$
$$1 \text{ rd.} = 16\frac{1}{2} \text{ or } \frac{33}{2} \text{ ft.}$$
$$7{,}360 \text{ rd.} = \overset{3680}{\cancel{7360}} \times \frac{33}{\cancel{2}} = 121{,}440 \text{ ft.}$$

121,440 ft. $\div$ 30 = 4,048 rails for one side of the track.

The number of rails for 2 sides of the track = 2 $\times$ 4,048, or 8,096 rails. Ans.

(106) 15 ft. 5 in. $\times$ 4 = 60 ft. 20 in., the length of the four pieces.

ft.	in.
60	20
14	8
8	10
82	38, or 85 ft. 2 in. = the length of the shaft.

15 ft.	5 in.
	4
60 ft.	20 in.

From the length of the shaft we must subtract 8 in. $\times$ 2 = 16 in. to get the distance between the end hangers.

ft.	in.
82	38
	16
82	22, or 83 ft. 10 in.

Since there are six hangers there are *five* spaces; the length of one space is 83 ft. 10 in. $\div$ 5 = 16 ft. $9\frac{1}{5}$ in. Ans.

(107) Since the distance around a wheel is $\frac{22}{7}$ times the distance across it, the distance around the wheel will equal (9 ft. $6\frac{1}{2}$ in.) $\times \frac{22}{7}$. We will reduce 9 ft. to inches before multiplying by the fraction. 9 ft. $\times$ 12 = 108 in. 108 in. + $6\frac{1}{2}$ in. = $114\frac{1}{2}$ in.

Reducing $114\frac{1}{2}$ to an improper fraction, we have $\frac{229}{2}$.

Multiplying, $\frac{229}{\cancel{2}} \times \frac{\overset{11}{\cancel{22}}}{7} = \frac{2,519}{7} = 359\frac{6}{7}$ in.

$$\begin{array}{r} 229 \\ \times\ 11 \\ \hline 229 \\ 229 \\ \hline 2519 \end{array}$$

Dividing $359\frac{6}{7}$ in. by 12 to reduce to feet, we have 29 ft. $11\frac{6}{7}$ in.

$$\begin{array}{l} 12)359\tfrac{6}{7} \text{ in. } (29 \text{ ft.} \\ \underline{24} \\ 119 \\ \underline{108} \\ 11\tfrac{6}{7} \text{ in.} \end{array}$$

(108) Reducing 18 ft. $11\frac{1}{4}$ in. to inches, we have $227\frac{1}{4}$ in., or 227.25 in.

$$\begin{array}{rl} \text{ft.} & \text{in.} \\ 18 & 11\tfrac{1}{4} \\ \underline{12} & \\ 36 & \\ \underline{18} & \\ 216 & \\ \underline{11\tfrac{1}{4}} & \\ 227\tfrac{1}{4} \text{ in.} & \end{array}$$

$1\frac{1}{8} \times 2$, or $2\frac{1}{4}$ in., for the two end rivets is deducted from the length, leaving 225 in., which is divided into equal spaces by the rivets.

$$\begin{array}{r} 227.25 \text{ in.} \\ -\quad 2.25 \text{ in.} \\ \hline 225 \text{ in.} \end{array}$$

The pitch of the rivets (or the distance between their centers) is $1\frac{1}{4}$ in., or 1.25 in.; hence, $225 \div 1.25 = 180$ spaces between the rivets. But, since there will be 1 more rivet than the number of spaces, the number of rivets required for this boiler shell will be $180 + 1 = 181$. Ans.

```
1.25)225.00(180
     125
     ----
     1000
     1000
     ----
        0
```

(109) (*a*) To find the second power of a number, we must multiply the number by itself once; that is, use the number twice as a factor. Thus, the second power of 108 is $108 \times 108 = 11,664$. Ans.

```
  108
  108
 ----
  864
108
-----
11664  Ans.
```

(*b*) The third power of 181.25 equals the number obtained by using 181.25 as a factor three times. Thus, the third power of 181.25 is $181.25 \times 181.25 \times 181.25 = 5,954,345.703125$. Ans.

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2 + 2 = 4$ decimal places in the first product.

Since there are 4 decimal places in the multiplicand, and 2 in the multiplier, there are $4 + 2 = 6$ decimal places in the final product.

```
          181.25
          181.25
          ------
          90625
         36250
        18125
      145000
      18125
      ----------
      32851.5625
          181.25
      ----------
      164257812 5
     657031250
    328515625
  2628125000
  328515625
  ----------------
  5954345.703125 Ans.
```

(*c*) The fourth power of 27.61 is the number obtained by using 27.61 as a factor four times. Thus, the fourth power of 27.61 is 27.61 × 27.61 × 27.61 × 27.61 = 581,119.73780641. Ans.

Since there are 2 decimal places in the multiplier, and 2 in the multiplicand, there are 2 + 2 = 4 decimal places in the first product.

Since there are 4 decimal places in the multiplicand, and 2 in the multiplier, there are 4 + 2 = 6 decimal places in the second product.

Since there are 6 decimal places in the multiplicand, and 2 in the multiplier, there are 6 + 2 = 8 decimal places in the final product.

```
          27.61
          27.61
          2761
        16566
       19327
       5522
       762.3121
          27.61
       7623121
     45738726
    53361847
   15246242
   21047.437081
          27.61
   21047437081
  1262846224 86
 147332059567
 42094874162
 581119.73780641
                 Ans.
```

(110) (*a*) $106^2 = 106 \times 106 = 11{,}236$. **Ans.**

```
  106
  106
  636
 106
 11236  Ans.
```

(*b*) $\left(182\frac{1}{8}\right)^2 = 182\frac{1}{8} \times 182\frac{1}{8} = 33{,}169.515625$. Ans.

$\frac{1}{8} = 8$) 1.000 (.125

```
 8
 20
 16
  40
  40
```

```
        182.125
        182.125
        910625
       364250
      182125
     364250
    1457000
    182125
    33169.515625  Ans.
```

Since there are 3 decimal places in the multiplier and 3 in the multiplicand, there are 3 + 3 = 6 decimal places in the product.

(*c*) $.005^2 = .005 \times .005 = .000025$. Ans.

```
    .005
    .005
 -------
 .000025   Ans.
```

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are $3 + 3 = 6$ decimal places in the product.

(*d*) $.0063^2 = .0063 \times .0063 = .00003969$. Ans.

```
     .0063
     .0063
     -----
       189
      378
 ---------
 .00003969   Ans.
```

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are $4 + 4 = 8$ decimal places in the product.

(*e*) $10.06^2 = 10.06 \times 10.06 = 101.2036$. Ans.

```
    10.06
    10.06
    -----
     6036
 1006
 --------
 101.2036   Ans.
```

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, there are $2 + 2 = 4$ decimal places in the product.

(*f*) $67.85^2 = 67.85 \times 67.85 = 4{,}603.6225$. Ans.

```
     67.85
     67.85
    ------
     33925
    54280
   47495
  40710
 ---------
 4603.6225   Ans.
```

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2 + 2 = 4$ decimal places in the product.

(*g*) $967{,}845^2 = 967{,}845 \times 967{,}845 = 936{,}723{,}944{,}025$. Ans.

```
          967845
          967845
          ------
         4839225
        3871380
       7742760
      6774915
     5807070
    8710605
    ------------
    936723944025   Ans.
```

(*h*) A fraction may be raised to any power by raising both numerator and denominator to the required power.

Thus, $\left(\frac{7}{16}\right)^2 = \frac{7}{16} \times \frac{7}{16} = \frac{7 \times 7}{16 \times 16} = \frac{49}{256}$. Ans.

(*i*) $\left(\frac{1}{4}\right)^2 = \frac{1}{4} \times \frac{1}{4} = \frac{1 \times 1}{4 \times 4} = \frac{1}{16}$. Ans.

(111) (*a*) $753^3 = 753 \times 753 \times 753 = 426{,}957{,}777$. Ans.

$$
\begin{array}{r}
753 \\
753 \\
\hline
2259 \\
3765 \\
5271 \\
\hline
567009 \\
753 \\
\hline
1701027 \\
2835045 \\
3969063 \\
\hline
426957777
\end{array}
$$
Ans.

(*b*) $987.4^3 = 987.4 \times 987.4 \times 987.4 = 962{,}674{,}279.624$. Ans.

$$
\begin{array}{r}
987.4 \\
987.4 \\
\hline
39496 \\
69118 \\
78992 \\
88866 \\
\hline
974958.76 \\
987.4 \\
\hline
389983504 \\
682471132 \\
779967008 \\
877462884 \\
\hline
962674279.624
\end{array}
$$
Ans.

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, there are $1 + 1 = 2$ decimal places in the first product.

Since there are 2 decimal places in the multiplicand and 1 in the multiplier, there are $2 + 1 = 3$ decimal places in the final product.

(*c*) $.005^3 = .005 \times .005 \times .005 = .000000125$. Ans.

$$
\begin{array}{r}
.005 \\
.005 \\
\hline
.000025 \\
.005 \\
\hline
.000000125
\end{array}
\quad \text{Ans.}
$$

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are $3 + 3 = 6$ decimal places in the first product; but, as there are only 2 figures in the product, we prefix four ciphers to make the necessary 6 decimal places.

Since there are 6 decimal places in the multiplicand and 3 in the multiplier, there are $6 + 3 = 9$ decimal places in the final product. In this case we prefix six ciphers to form the necessary 9 decimal places.

(*d*) $.4044^3 = .4044 \times .4044 \times .4044 = .066135317184$. Ans.

$$
\begin{array}{r}
.4044 \\
.4044 \\
\hline
16176 \\
16176 \\
16176 \\
\hline
.16353936 \\
.4044 \\
\hline
65415744 \\
65415744 \\
65415744 \\
\hline
.066135317184
\end{array}
\quad \text{Ans.}
$$

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are $4 + 4 = 8$ decimal places in the first product.

Since there are 8 decimal places in the second multiplicand and 4 in the multiplier, there are $8 + 4 = 12$ decimal places in the final product; but, as there are only 11 figures in the product, we prefix 1 cipher to make the necessary 12 decimal places.

(*e*) $.0133^3 = .0133 \times .0133 \times .0133 = .000002352637$. Ans.

$$\begin{array}{r} .0133 \\ .0133 \\ \hline 399 \\ 399 \\ 133 \\ \hline .00017689 \\ .0133 \\ \hline 53067 \\ 53067 \\ 17689 \\ \hline .000002352637 \end{array}$$ Ans.

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, we should point off $4 + 4 = 8$ decimal places in the product; but, as there are only 5 figures in the product, we prefix three ciphers to form the eight necessary decimal places in the first product.

Since there are 8 decimal places in the multiplicand and 4 in the multiplier, we should point off $8 + 4 = 12$ decimal places in the product; but, as there are only 7 figures in the product, we prefix 5 ciphers to make the 12 necessary decimal places in the final product.

(*f*) $301.011^3 = 301.011 \times 301.011 \times 301.011 =$ 27,273,890.942264331. Ans.

$$\begin{array}{r} 301.011 \\ 301.011 \\ \hline 301011 \\ 301011 \\ 301011 \\ 903033 \\ \hline 90607.622121 \\ 301.011 \\ \hline 90607622121 \\ 90607622121 \\ 90607622121 \\ 271822866363 \\ \hline 27273890.942264331 \end{array}$$ Ans.

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, we should point off $3 + 3 = 6$ decimal places in the first product.

Since there are 6 decimal places in the multiplicand and 3 in the multiplier, we should point off $6 + 3 = 9$ decimal places in the final product.

(*g*) $\left(\frac{1}{8}\right)^3 = \frac{1}{8} \times \frac{1}{8} \times \frac{1}{8} = \frac{1 \times 1 \times 1}{8 \times 8 \times 8} = \frac{1}{512}$. Ans.

$$\begin{array}{r} 8 \\ \times\ 8 \\ \hline 64 \\ \times\ 8 \\ \hline 512 \end{array}$$

(*h*) To find any power of a mixed number, first reduce it to an improper fraction, and then multiply the numerators together for the numerator of the answer and multiply the denominators together for the denominator of the answer.

$$\left(3\tfrac{3}{4}\right)^3 = \frac{15}{4} \times \frac{15}{4} \times \frac{15}{4} = \frac{15 \times 15 \times 15}{4 \times 4 \times 4} = \frac{3,375}{64} = 52.734375.$$

Ans

$$3\tfrac{3}{4} = \frac{(3 \times 4) + 3}{4} = \frac{12 + 3}{4} = \frac{15}{4}.$$

$$\begin{array}{r} 15 \\ 15 \\ \hline 75 \\ 15 \\ \hline 225 \\ 15 \\ \hline 1125 \\ 225 \\ \hline 3375 \end{array}$$

$$\begin{array}{l} 64\,)\,3375.000000\,(\,52.734375 \\ 320 \\ \hline 175 \\ 128 \\ \hline 470 \\ 448 \\ \hline 220 \\ 192 \\ \hline 280 \\ 256 \\ \hline 240 \\ 192 \\ \hline 480 \\ 448 \\ \hline 320 \\ 320 \\ \hline \end{array}$$

Since six ciphers were annexed to the dividend, six decimal places must be pointed off in the quotient.

(112) $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32.$ Ans

$$\begin{array}{r} 2 \\ \underline{2} \\ 4 \\ \underline{2} \\ 8 \\ \underline{2} \\ 16 \\ \underline{2} \\ 32 \end{array} \text{ Ans.}$$

(113) $3^4 = 3 \times 3 \times 3 \times 3 = 81.$ Ans.

$$\begin{array}{r} 3 \\ \underline{3} \\ 9 \\ \underline{3} \\ 27 \\ \underline{3} \\ 81 \end{array} \text{ Ans.}$$

(114) $7^9 = 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 =$ 40,353,607. Ans

$$\begin{array}{r} 7 \\ \underline{7} \\ 49 \\ \underline{7} \\ 343 \\ \underline{7} \\ 2401 \\ \underline{7} \\ 16807 \\ \underline{7} \\ 117649 \\ \underline{7} \\ 823543 \\ \underline{7} \\ 5764801 \\ \underline{7} \\ 40353607 \end{array} \text{ Ans.}$$

(115) (*a*) $1.2^4 = 1.2 \times 1.2 \times 1.2 \times 1.2 = 2.0736$. Ans.

$$\begin{array}{r} 1.2 \\ \underline{1.2} \\ 24 \\ \underline{12} \\ 1.44 \\ \underline{1.2} \\ 288 \\ \underline{144} \\ 1.728 \\ \underline{1.2} \\ 3456 \\ \underline{1728} \\ 2.0736 \end{array}$$

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we should point off $1 + 1 = 2$ decimal places in the first product.

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we should point off $2 + 1 = 3$ decimal places in the second product.

Since there are 3 decimal places in the multiplicand and 1 in the multiplier, we should point off $3 + 1 = 4$ decimal places in the final product.

(*b*) $11^5 = 11 \times 11 \times 11 \times 11 \times 11 = 161{,}051$. Ans.

$$\begin{array}{r} 11 \\ \underline{11} \\ 11 \\ \underline{11} \\ 121 \\ \underline{11} \\ 121 \\ \underline{121} \\ 1331 \\ \underline{11} \\ 1331 \\ \underline{1331} \\ 14641 \\ \underline{11} \\ 14641 \\ \underline{14641} \\ 161051 \end{array}$$

(*c*) $1^6 = 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$. Ans.

(*d*) $.01^4 = .01 \times .01 \times .01 \times .01 = .00000001$. Ans.

$$\begin{array}{r} .01 \\ .01 \\ \hline .0001 \\ .01 \\ \hline .000001 \\ .01 \\ \hline .00000001 \end{array}$$

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, we should point off $2 + 2 = 4$ decimal places in the first product; but, as there is only 1 figure in the product, we prefix 3 ciphers to make the necessary 4 decimal places.

Since there are 4 decimal places in the second multiplicand and 2 in the multiplier, we should point off $4 + 2 = 6$ decimal places in the second product. It is necessary to prefix 5 ciphers to make six decimal places.

Since there are 6 decimal places in the third multiplicand and 2 in the multiplier, we should point off $6 + 2 = 8$ decimal places in the product. It is necessary to prefix 7 ciphers to make 8 decimal places in the final product.

(*e*) $.1^5 = .1 \times .1 \times .1 \times .1 \times .1 = .00001$. Ans.

$$\begin{array}{r} .1 \\ .1 \\ \hline .01 \\ .1 \\ \hline .001 \\ .1 \\ \hline .0001 \\ .1 \\ \hline .00001 \end{array}$$

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we should point off $1 + 1 = 2$ decimal places in the first product. It is necessary to prefix 1 cipher to the product.

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we should point off $2 + 1 = 3$ decimal places in the second product. It is necessary to prefix 2 ciphers to the second product.

Since there are 3 decimal places in the third multiplicand and 1 in the multiplier, we should point off $3 + 1 = 4$ decimal places in the third product. We prefix three ciphers to this product.

Since there are 4 decimal places in the fourth multiplicand and 1 in the multiplier, we should point off $4 + 1$ or 5 decimal places in the final product. It is necessary to prefix 4 ciphers to this product.

(116) Evolution is the reverse of involution. In involution we find the *power* of a number by multiplying the number by itself one or more times, while in evolution we find the *number* or *root* which was multiplied by itself one or more times to make the power.

(117) (*a*)

(*a*) 1	$\sqrt{3'48'67'84.40'10} = 1867.29+$ Ans.
1	(*b*) 1
(*d*) 20	(*c*) 248
8	224
28	(*e*) 2467
8	2196
360	27184
6	26089
366	109540
6	74684
3720	3485610
7	3361041
3727	124569
7	
37340	
2	
37342	
2	
373440	
9	
373449	

EXPLANATION.—(1) Divide the power into periods. In the above case, where the power consists of a whole number and a decimal, we begin at the decimal point and proceed to the *left* in pointing off the *whole* number, and towards the *right* in pointing off the *decimal*, annexing a 0 to complete the last decimal period. In *square root two* figures constitute a period.

Find the greatest single number whose square is *less* than or *equal* to 3, the first period. This is evidently 1, since $2^2 = 4$, which is greater than 3. Write 1 as the first figure of the root; also, write it to the left, as shown at (*a*). Now, multiply the 1 at (*a*) by the 1 in the root, and write the result under the first period, or 3, as shown at (*b*). Sub-

tract, and bring down the next period 48, and annex it to the remainder 2, thus making 248 the *dividend*, as shown at (*c*). Add the root already found to the 1 at (*a*), thereby obtaining 2, and annex a cipher to this 2, thus making it 20, which we call the *trial divisor.*

Divide the dividend 248 at (*c*) by the trial divisor 20 at (*d*) and obtain 8, which is probably the next figure of the root. Write 8 in the root, as shown, and also add it to 20, the trial divisor, making it 28. This is called the *complete divisor.*

(2) Multiply the complete divisor 28 by 8, the second figure of the root, and subtract the result from the dividend (*c*). The remainder is 24, to which annex the next period making 2,467, as shown at (*e*), which call the *new dividend.*

Add the second figure of the root, or 8, to the divisor 28, and annex a cipher, thus obtaining 360. Dividing 2,467 by 360, we find 6 to be the next figure of the root. Adding this last figure of the root, or 6, to 360, we get 366, which, multiplied by this last figure of the root, or 6, gives 2,196, which we write under 2,467 and subtract.

(3) Annexing the next period 84 to the remainder 271 gives 27,184 as the next new dividend. Now, adding the third figure of the root to 366, and annexing a cipher, as before, we have 3,720. Dividing 27,184 by 3,720, the result is 7, which write as the next figure of the root. Adding the fourth figure of the root, or 7, to 3,720, we get 3,727, which, multiplied by 7 of the root, gives 26,089, which write under 27,184 and subtract, obtaining 1,095 as a remainder.

(4) Annexing the next period, or 40, to the remainder 1,095 gives 109,540 as the next new dividend. Adding the last figure of the root to 3,727, and annexing a cipher, as before, the result is 37,340. Dividing 109,540 by 37,340, the result is 2, which write as the next figure of the root. Adding the fifth figure of the root, or 2, to 37,340, we obtain 37,342, which, multiplied by 2 of the root, gives 74,684, which write under 109,540 and subtract, obtaining 34,856 as a remainder.

(5) Annexing the next period, or 10, to the remainder

34,856 gives 3,485,610 as the next new dividend. Now, adding the last figure of the root to 37,342, and annexing a cipher, as before, the result is 373,440. Dividing 3,485,610 by 373,440 gives 9 as a result, which write as the next, or last figure, of the root. Adding the last figure of the root, or 9, to 373,440, we get 373,449, which, multiplied by 9 of the root, gives 3,361,041, which write under 3,485,610 and subtract. Since there is a remainder, we know the given power is not a perfect square, so we place + after the root.

In this problem there are *six* periods—four in the whole number and two in the decimal—hence, there will be *six* figures in the root (since we obtain one figure of the root for each period), four figures constituting the whole number and two figures the decimal of the root. Hence,

$$\sqrt{3,486,784.4010} = 1,867.29 +.$$

```
(b)  (a)  3        √9'00'00'99.40'09'00 = 3000.016 +  Ans.
          3   (b)   9
         ---      ------------
(d)      60  (c)    0000994009
          0               600001
         ---              --------
         600              39400800
           0              36000156
         ----             --------
         6000              3400644
            0
         -----
         60000
             0
         ------
         600000
              1
         ------
         600001
              1
         -------
         6000020
              6
         -------
         6000026
```

EXPLANATION.—Beginning at the decimal point, we point off the whole number into periods of *two* figures each, proceeding from *right* to *left;* also, point off the decimals into periods of *two* figures each, proceeding from *left* to *right.* The largest number whose square is contained in the first period, 9, is 3; hence, 3 is the first figure of the root. Place 3 at the left, as shown at (*a*), and multiply it by the first

figure in the root, or 3. The result is 9. Write 9 under the first period, 9, as at (*b*), subtract, and there is no remainder. Bring down the next period, which is 00, as shown at (*c*). Add the root already found to the 3 at (*a*), obtaining 6, and annex a cipher to this 6, thus making it 60, which is the *trial divisor*, as shown at (*d*). Divide the dividend (*c*) by the trial divisor and obtain 0 as the next figure in the root. Write 0 in the root as shown, and also add it to the trial divisor 60, and annex a cipher, thereby making the next trial divisor 600. Bring down the next period, 00, annex it to the dividend already obtained and divide it by the trial divisor. 600 is contained in 0000, 0 times, so we place another cipher in the root. Write 0 in the root, as shown, and also add it to the trial divisor 600, and annex a cipher, thereby making the next trial divisor 6,000. Bring down the next period, 99. The trial divisor 6,000 is contained in 000099, 0 times, so we place 0 as the next figure in the root, as shown, and also add it to the trial divisor 6,000, and annex a cipher, thereby making the next trial divisor 60,000. Bring down the next period, 40, and annex it to the dividend already obtained to form the new dividend, 00009940, and divide it by the trial divisor 60,000. 60,000 is contained in 00009940, 0 times, so we place another cipher in the root, as shown, and also add it to the trial divisor 60,000, and annex one cipher, thereby making the next trial divisor 600,000. Bring down the next period, 09, and annex it to the dividend already obtained to form the new dividend, 0000994009, and divide it by the trial divisor 600,000. 600,000 is contained in 0000994009 once, so we place 1 as the next figure in the root, and also add it to the trial divisor 600,000, thereby making the complete divisor 600,001. Multiply the complete divisor 600,001 by 1, the sixth figure in the root, and subtract the result obtained from the dividend. The remainder is 394,008, to which we annex the next period, 00, to form the next new dividend, or 39,400,800. Add the sixth figure of the root, or 1, to the divisor 600,001, and annex a cipher, thus obtaining 6,000,020 as the next trial divisor. Dividing 39,400,800 by 6,000,020, we find 6 to be the next figure of the root. Adding this last

figure, 6, to the trial divisor, we obtain 6,000,026 for our next complete divisor, which, multiplied by the last figure of the root, or 6, gives 36,000,156, which write under 39,400,800 and subtract. Since there is a remainder, it is clearly evident that the given power is not a perfect square, so we place + after the root.

In this problem, there are *seven* periods—four in the whole number and three in the decimal; hence, there will be *seven* figures in the root, *four* figures constituting the whole number and three figures the decimal of the root. Hence, $\sqrt{9{,}000{,}099.4009} = 3{,}000.016+$. Ans.

(*c*)

$$\begin{array}{r} 3 \\ \underline{3} \\ 60 \\ \underline{5} \\ 65 \end{array} \qquad \begin{array}{l} \sqrt{.00'12'25} = .035. \quad \text{Ans.} \\ \quad 00 \\ \quad \overline{\quad 12} \\ \quad\quad 9 \\ \quad \overline{\quad 325} \\ \quad\quad 325 \\ \quad \overline{\qquad} \end{array}$$

Pointing of periods, we find that the first period is composed of ciphers; hence, the first figure of the root will be a cipher. No further explanation is necessary, since this problem is solved in a manner exactly similar to the problem given in Art. **255.** Since there are *three* decimal periods in the power, there will be three decimal figures in the root.

(*d*)

$$\begin{array}{r} 1 \\ \underline{1} \\ 20 \\ \underline{0} \\ 200 \\ \underline{3} \\ 203 \\ \underline{3} \\ 2060 \\ \underline{9} \\ 2069 \end{array} \qquad \begin{array}{l} \sqrt{1'07'95.21} = 103.9 \quad \text{Ans.} \\ 1 \\ \overline{\quad 0795} \\ \quad\; 609 \\ \overline{\quad 18621} \\ \quad 18621 \\ \overline{\qquad} \end{array}$$

(e)

2	$\sqrt{7'30'08.05} = 270.2+$ Ans.
2	4
40	330
7	329
47	10805
7	10804
5400	1
2	
5402	

(f) $\sqrt{9} = 3.$ Ans.

(g)

9	$\sqrt{.90'00'00} = .948+$ Ans
9	81
180	900
4	736
184	16400
4	15104
1880	1296
8	
1888	

In Art. **258,** we see that every period of a decimal must consist of two figures when extracting the square root. We must, therefore, annex one cipher to .9 to form the first period.

(**118**) (a)

(1)	(2)	
6	36	$\sqrt[3]{.327'680'000} = .689+$ Ans.
6	72	216
12	10800	111680
6	1504	98432
180	12304	13248000
8	1568	12650769
188	1387200	597231
8	18441	
196	1405641	
8		
2040		
9		
2049		

EXPLANATION.—(1) When extracting the *cube* root, we divide the power into periods of three figures each. Always begin at the decimal point and proceed to the *left* in pointing off the whole number, and to the *right* in pointing off the decimal. In this power, $\sqrt[3]{.32768}$, a cipher must be annexed to 68 to complete the decimal period. Cipher periods may now be annexed until the root has as many figures as desired.

(2) We find by trial that the largest number whose cube is contained in the first period 327 is 6. Write 6 as the first figure of the root, also at the extreme left at the head of column (1). Multiply the 6 in column (1) by the first figure of the root 6, and write the product 36 at the head of column (2). Multiply the number in column (2) by the first figure of the root 6, and write the product 216 under the figures in the first period. Subtract and bring down the next period 680, annexing it to the remainder 111, thereby obtaining 111,680 for a new dividend. Add the first figure of the root 6 to the number in column (1), obtaining 12, which we call the *first correction;* multiply the first correction 12 by the first figure of the root and we obtain 72 as the product, which, added to 36 of column (2), gives 108. Add the first figure of the root to the first correction, and we obtain 18 as the *second correction.* To this annex *one* cipher, and annexing two ciphers to 108, we have 10,800 for the trial divisor. Dividing the dividend by the trial divisor, we see that it is contained about 8 times, so we write 8 as the second figure of the root. Adding the second figure of the root to 180, we obtain 188. This, multiplied by the second figure of the root 8, equals 1,504, which, added to the trial divisor 10,800, forms the *complete divisor*, 12,304. Multiplying the complete divisor 12,304 by 8, the second figure of the root, gives 98,432. Write 98,432 under the dividend 111,680; subtract, and there is a remainder of 13,248. To this remainder annex the next period, 000, thereby obtaining 13,248,000 for the next new dividend.

(3) Adding the second figure of the root 8 to the number in column (1), 188, we have 196 for the *first new correction.* This, multiplied by the second figure of the root 8, gives 1,568. Adding this product to the last complete divisor gives 13,872. Adding the second figure of the root, 8, to the first new correction, 196, we obtain 204 for the *second new correction.* Annexing one cipher to 204 and two ciphers to 13,872, we obtain 1,387,200 for the new trial divisor. Dividing the dividend by the trial divisor 1,387,200, we see that it is contained about 9 times. Write 9 as the third figure of the root. Add the third figure of the root, 9, to the last number in column (1), 2,040, thereby obtaining 2,049. This, multiplied by 9, the third figure of the root, equals 18,441, which, added to the trial divisor 1,387,200, forms the complete divisor 1,405,641. Multiplying the complete divisor by the third figure of the root 9, and subtracting, we have a remainder of 597,231. The answer, then, is .689 +. There are as many decimal places in the root as there are decimal periods in the power, or three decimal places, in this case.

(*b*)

(1)	(2)
4	16
4	32
8	4800
4	244
120	5044
2	
122	

$\sqrt[3]{74'088} = 42$ Ans.

```
 64
 10088
 10088
```

(*c*)

45.2115+ Ans.

$\sqrt[3]{92'416.000'000'000'000} =$

```
64
-----
28416
27125
--------
 1291000
 1220408
---------
   70592000
   61304761
-----------
    9287239000
    6131967931
--------------
    3155271069000
    3066085689875
----------------
       89185379125
```

(1)

```
    4
    4
    -
    8
    4
  ---
  120
    5
  ---
  125
    5
  ---
  130
    5
 ----
 1350
    2
 ----
 1352
    2
 ----
 1354
    2
-----
13560
    1
-----
13561
    1
-----
13562
    1
------
135630
     1
------
135631
     1
------
135632
     1
-------
1356330
      5
-------
1356335
```

(2)

```
16
32
----
4800
 625
----
5425
 650
------
607500
  2704
------
610204
  2708
--------
61291200
   13561
--------
61304761
   13562
----------
6131832300
    135631
----------
6131967931
    135632
------------
613210356300
     6781675
------------
613217137975
```

Since *four cipher periods* were annexed, we must point off *four* decimal figures in the root.

(*d*)

(1)	(2)
7	49
7	98
14	14700
7	424
210	15124
2	
212	

$\sqrt[3]{.373'248} = .72$ Ans

343

30248

30248

(*e*)

(1)	(2)
1	1
1	2
2	300
1	64
30	364
2	68
32	4320000
2	25249
34	4345249
2	
3600	
7	
3607	

$\sqrt[3]{1'758.416'743} = 12.07$ Ans.

1

758

728

30416743

30416743

(*f*)

(1)	(2)
1	1
1	2
2	30000
1	1836
300	31836
6	
306	

$\sqrt[3]{1'191'016} = 106$ Ans

1

191016

191016

(*g*) In Arts. **279** and **280,** we find that when the given number is in the form of a fraction, and it is required to find some root of it, the simplest method is to reduce the fraction to its equivalent decimal and then extract the required root of the decimal.

Thus, $\sqrt[3]{\frac{4}{32}} = ?$ $\frac{4}{32} = .125$, since

$$\begin{array}{r} 32\,)\,4.000\,(\,.125 \\ \underline{32} \\ 80 \\ \underline{64} \\ 160 \\ \underline{160} \end{array}$$

(1)	(2)	$\sqrt[3]{.125} = .5.$
5	25	125

Hence, $\sqrt[3]{\frac{4}{32}} = .5.$ Ans.

(*h*) (See Art. **279.**)

$\sqrt[3]{\frac{27}{256}} = ?$ $\frac{27}{256} = .10546875$, since

$$\begin{array}{l} 256\,)\,27.00000000\,(\,.10546875 \\ \underline{256} \\ 1400 \\ \underline{1280} \\ 1200 \\ \underline{1024} \\ 1760 \\ \underline{1536} \\ 2240 \\ \underline{2048} \\ 1920 \\ \underline{1792} \\ 1280 \\ \underline{1280} \end{array}$$

(1)	(2)	
4	16	$\sqrt[3]{.105'468'750} = .472+$ Ans.
4	32	64
8	4800	41468
4	889	39823
120	5689	1645750
7	938	1331048
127	662700	314702
7	2824	
134	665524	
7		
1410		
2		
1412		

(119)

(1)	(2)	
1	1	$\sqrt[3]{2.000'000'000'000} = 1.2599+$
1	2	1 Ans.
2	300	1000
1	64	728
30	364	272000
2	68	225125
32	43200	46875000
2	1825	42491979
34	45025	4383021000
2	1850	4282771599
360	4687500	100249401
5	33831	
365	4721331	
5	33912	
370	475524300	
5	340011	
3750	475864311	
9		
3759		
9		
3768		
9		
37770		
9		
37779		

In this problem it was necessary to annex *four cipher periods*, in order to carry the decimal of the root to the required number of decimal places, or four.

(120)

(1)	(2)
1	1
1	2
2	300
1	136
30	436
4	152
34	58800
4	1696
38	60496
4	1712
420	6220800
4	8644
424	6229444
4	
428	
4	
4320	
2	
4322	

$\sqrt[3]{3.000'000'000} = 1.442+$ **Ans.**

1
2000
1744
256000
241984
14016000
12458888
1557112

In this problem it was necessary to annex *three* cipher periods, in order to carry the decimal of the root to the required number of decimal places, or three.

(121)

(*a*)

$\sqrt{1'23.21} = 11.1$ Ans.

1	1
1	23
20	21
1	221
21	221
1	
220	
1	
221	

(*b*)

$\sqrt{1'14.92'10} = 10.72+$ Ans

1	1
1	1492
200	1449
7	4310
207	4284
7	26
2140	
2	
2142	

(*c*)

7	$\sqrt{50'26'81} = 709$ Ans.
7	49
140	12681
0	12681
1400	
9	
1409	

(*d*)

2	$\sqrt{.00'04'12'09} = .0203$ Ans.
2	00
400	04
3	4
403	1209
	1209

(122) (*a*)

(1)	(2)	$\sqrt[3]{.006'500'000} = .186+$ Ans.
1	1	1
1	2	5500
2	300	4832
1	304	668000
30	604	602856
8	368	65144
38	97200	
8	3276	
46	100476	
8		
540		
6		
546		

(*b*)

(1)	(2)	$\sqrt[3]{.021'000} = .27+$ Ans
2	4	8
2	8	13000
4	1200	11683
2	469	1317
60	1669	
7		
67		

(c)

(1)	(2)	$\sqrt[3]{8'036'054'027} = 2003$ Ans
2	4	8
2	8	036054027
4	12000000	36054027
2	18009	
6000	12018009	
3		
6003		

(d)

(1)	(2)	$\sqrt[3]{.000'004'096} = .016$ Ans.
1	1	000
1	2	004
2	300	1
1	216	3096
30	516	3096
6		
36		

(e)

(1)	(2)	$\sqrt[3]{17.000'000} = 2.57+$ Ans.
2	4	8
2	8	9000
4	1200	7625
2	325	1375000
60	1525	1349593
5	350	25407
65	187500	
5	5299	
70	192799	
5		
750		
7		
757		

(123) (*a*) $\sqrt{\frac{1,225}{5,476}} = \frac{\sqrt{1,225}}{\sqrt{5,476}}$.

$$\begin{array}{r} 3 \\ 3 \\ \hline 60 \\ 5 \\ \hline 65 \end{array} \qquad \begin{array}{l} \sqrt{12'25} = 35 \\ 9 \\ \hline 325 \\ 325 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ 7 \\ \hline 140 \\ 4 \\ \hline 144 \end{array} \qquad \begin{array}{l} \sqrt{54'76} = 74 \\ 49 \\ \hline 576 \\ 576 \\ \hline \end{array}$$

Hence, $\sqrt{\frac{1,225}{5,476}} = \frac{35}{74}$. Ans.

(*b*)

$$\begin{array}{r} 5 \\ 5 \\ \hline 100 \\ 8 \\ \hline 108 \end{array} \qquad \begin{array}{l} \sqrt{.33'64} = .58 \\ 25 \quad \text{Ans.} \\ \hline 864 \\ 864 \\ \hline \end{array}$$

(*c*)

$$\begin{array}{r} 3 \\ 3 \\ \hline 60 \\ 1 \\ \hline 61 \\ 1 \\ \hline 620 \\ 6 \\ \hline 626 \\ 6 \\ \hline 6320 \\ 2 \\ \hline 6322 \\ 2 \\ \hline 63240 \\ 2 \\ \hline 63242 \end{array} \qquad \begin{array}{l} \sqrt{.10'00'00'00'00} = .31622+ \quad \text{Ans.} \\ 9 \\ \hline 100 \\ 61 \\ \hline 3900 \\ 3756 \\ \hline 14400 \\ 12644 \\ \hline 175600 \\ 126484 \\ \hline 49116 \end{array}$$

(*d*) $25.0\frac{3}{4} = 25.075.$

```
5            √25.07'50'00'00'00 = 5.00749 +
5             25                      Ans.
10000          075000
    7           70049
10007            495100
    7            400576
100140            9452400
     4            9013401
100144             438999
     4
1001480
      9
1001489
```

(*e*) $.000\frac{4}{9} = .0004444444 +.$

```
2         √.00'04'44'44'44 = .02108+   Ans
2          00
40          04
 1           4
41            44
 1            41
4200           34444
   8           33664
4208             780
```

(124) $11.7 : 13 :: 20 : x$
$11.7x = 13 \times 20$
$11.7x = 260$

The product of the means equals the product of the extremes.

$$x = \frac{260}{11.7}$$

```
11.7)260.000(22.22+   Ans
     234
      260
      234
       260
       234
        260
        234
         26
```

(125) (*a*) $20 + 7 : 10 + 8 :: 3 : x.$

$$27 : 18 :: 3 : x$$
$$27x = 18 \times 3$$
$$27x = 54$$
$$x = \frac{54}{27} = 2. \quad \text{Ans.}$$

(*b*) $(12)^2 : (100)^2 :: 4 : x$

$$144 : 10{,}000 :: 4 : x$$
$$144x = 10{,}000 \times 4$$
$$144x = 40{,}000$$

$$\begin{array}{r} 12 \\ 12 \\ \hline 144 \end{array} \qquad \begin{array}{r} 100 \\ 100 \\ \hline 10000 \end{array}$$

$$x = \frac{40{,}000}{144}$$

$$\begin{array}{l} 144)40000.0(277.7+ \quad \text{Ans.} \\ \quad 288 \\ \quad \overline{1120} \\ \quad 1008 \\ \quad \overline{\ \ 1120} \\ \quad\ \ 1008 \\ \quad \overline{\ \ \ \ 1120} \\ \quad\ \ \ \ 1008 \\ \quad\ \ \ \ \overline{} \end{array}$$

(126) (*a*) $\frac{4}{x} = \frac{7}{21}$ is equivalent to $4 : x :: 7 : 21$. The product of the means equals the product of the extremes. Hence,

$$7x = 4 \times 21$$
$$7x = 84$$
$$x = \frac{84}{7}, \text{ or } 12. \quad \text{Ans.}$$

In like manner,

(*b*) $\frac{x}{24} = \frac{8}{16}$ is equivalent to $x : 24 :: 8 : 16.$

$$16x = 24 \times 8$$
$$16x = 192$$
$$x = \frac{192}{16} = 12. \quad \text{Ans.}$$

(*c*) $\frac{2}{10} = \frac{x}{100}$ is equivalent to $2 : 10 :: x = 100$

$$10x = 2 \times 100$$
$$10x = 200$$
$$x = \frac{200}{10} = 20. \quad \text{Ans.}$$

(*d*) $\frac{15}{45} = \frac{60}{x}$ is equivalent to $15 : 45 :: 60 : x$

$$15x = 45 \times 60$$
$$15x = 2{,}700$$
$$x = \frac{2{,}700}{15} = 180. \quad \text{Ans.}$$

(*e*) $\frac{10}{150} = \frac{x}{600}$ is equivalent to $10 : 150 :: x : 600$.

$$150x = 10 \times 600$$
$$150x = 6{,}000$$
$$x = \frac{6{,}000}{150} = 40. \quad \text{Ans}$$

(127) $x : 5 :: 27 : 12.5$

$$\begin{array}{r} 5 \\ 12.5 \,)\, 135.0 \,(\, 10\frac{4}{5}. \quad \text{Ans.} \\ 125 \\ \hline \frac{100}{125} = \frac{4}{5}. \end{array}$$

Hence, $x = 10\frac{4}{5}$. Ans.

(128) $45 : 60 :: x : 24$

$$60x = 45 \times 24$$
$$60x = 1{,}080$$
$$x = \frac{1{,}080}{60} = 18. \quad \text{Ans.}$$

(129) $x : 35 :: 4 : 7$

$$7x = 35 \times 4$$
$$7x = 140$$
$$x = \frac{140}{7} = 20. \quad \text{Ans.}$$

(130) $9 : x :: 6 : 24$

$$6x = 9 \times 24$$
$$6x = 216$$
$$x = \frac{216}{6} = 36. \quad \text{Ans.}$$

(131) $\sqrt[3]{1,000} : \sqrt[3]{1,331} :: 27 : x$ $\qquad \sqrt[3]{1,000} = 10$

$\sqrt[3]{1,331} = 11$

$10 : 11 :: 27 : x$

$10x = 297$

$x = \frac{297}{10} = 29.7.$ Ans.

(1)	(2)	
1	1	1'331(11
1	2	1
2	300	331
1	31	331
30	331	
1		
31		

(132) $64 : 81 :: 21^2 : x^2$. Extracting the square root of each term of any proportion does not change its value, so we find that $\sqrt{64} : \sqrt{81} :: \sqrt{21^2} : \sqrt{x^2}$ is the same as

$$8 : 9 :: 21 : x$$
$$8x = 189$$
$$x = 23.625. \quad \text{Ans.}$$

(133) $7 + 8 : 7 :: 30 : x$ is equivalent to $15 : 7 :: 30 : x$.

$$15x = 7 \times 30$$
$$15x = 210$$
$$x = \frac{210}{15} = 14. \quad \text{Ans.}$$

(134) $3\frac{1}{2}$ ft. = 3.5 ft., since $\frac{1}{2} = .5$.

$6\frac{3}{4}$ ft. = 6.75 ft., since $\frac{3}{4} = .75$.

Consider the question "What do we wish to find?" In this case it is "pounds." We know that a piece of shafting 3.5 ft. long weighs 37.45 lb., and we wish to know the weight of a piece of shafting $6\frac{3}{4}$ ft. long. It is evident that the weight of a piece of shafting 6.75 ft. long bears the same relation to the weight of a piece 3.5 ft. long that 6.75 ft. bears to 3.5 ft. Letting x occupy any place in the propor-

tion, we have the following, the value of x being the same in each. Thus,

(*a*) 3.5 ft. : 6.75 ft. :: 37.45 lb. : x lb.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb. Ans.}$$

(*b*) 6.75 ft. : 3.5 ft. :: x lb. : 37.45 lb.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb. Ans.}$$

(*c*) x lb. : 37.45 lb. :: 6.75 ft. : 3.5 ft.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb. Ans.}$$

(*d*) 37.45 lb. : x lb. :: 3.5 ft. : 6.75 ft.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb. Ans.}$$

(135) In this problem we are required to find "degrees." In examples of this kind, it is better to write the statement down as a simple direct proportion, and then make what changes are necessary. Written as a direct proportion, we would have 6 : 12 :: 24 : x. But, since the heat varies *inversely*, the proportion is an inverse one. Hence, inverting one of the couplets, we have 12 : 6 :: 24 : x. The example also states that the effect of the heat upon the thermometer varies inversely as the *square of the distance* from the burning body. Consequently, the two distances, 6 ft. and 12 ft., in the above proportion must be squared, and we have $12^2 : 6^2 :: 24 : x$. $12^2 = 144$ and $6^2 = 36$. Therefore, 144 : 36 :: 24 : x,

$$\text{or } x = \frac{36 \times 24}{144} = 6°. \text{ Ans.}$$

(136) This problem may be solved by very simple reasoning, thus: If sound travels at the rate of 6,160 feet in $5\frac{1}{2}$ seconds, in 1 minute, which is $10\frac{10}{11}$ times $5\frac{1}{2}$ seconds

$\left(60 \div \frac{11}{2} = 60 \times \frac{2}{11} = \frac{120}{11} = 10\frac{10}{11}\right)$, it will travel $10\frac{10}{11} \times$ 6,160 ft., or 67,200 ft. Ans.

$$\begin{array}{r} 6160 \\ 10\frac{10}{11} \\ \hline 61600 \\ 5600 \\ \hline 67200 \text{ ft. Ans.} \end{array} \qquad \frac{10}{11} \times 6,160 = \frac{61,600}{11} = 5,600.$$

Or, if sound travels at the rate of 6,160 feet in $5\frac{1}{2}$ seconds, in 1 second it will travel as many feet as $5\frac{1}{2}$ (5.5) is contained times in 6,160 feet, or 1,120 feet, and in 1 minute, or 60 seconds, it will travel $1,120 \times 60$, or 67,200 feet.

Or, it may be solved by proportion, as follows: In this problem it states that sound travels at the rate of 6,160 feet in $5\frac{1}{2}$ (5.5) seconds. We wish to know how far it will travel in 1 minute, which is equivalent to 60 seconds. It is evident that the distance it travels in 1 minute, or 60 seconds, bears the same relation to the distance it travels in 5.5 seconds that 1 minute, or 60 seconds, bears to 5.5 seconds. Letting x occupy any place in the proportion, we have the following, the value of x being the same in each. Thus,

(*a*) 6,160 feet : x feet :: 5.5 seconds : 60 seconds,

or $x = \frac{6,160 \times 60}{5.5} = \frac{369,600}{5.5}$

$$\begin{array}{l} 5.5)369600.0(67200 \text{ ft. Ans.} \\ \quad\;\; 330 \\ \hline \quad\;\;\; 396 \\ \quad\;\;\; 385 \\ \hline \quad\;\;\;\; 110 \\ \quad\;\;\;\; 110 \\ \hline \end{array}$$

(*b*) x feet : 6,160 feet :: 60 seconds : 5.5 seconds,

$$\text{or } x = \frac{6,160 \times 60}{5.5} = 67,200 \text{ feet. Ans.}$$

(*c*) **60 seconds : 5.5 seconds :: x feet : 6,160 feet,**

$$\text{or } x = \frac{6{,}160 \times 60}{5.5} = 67{,}200 \text{ feet. Ans.}$$

(*d*) **5.5 seconds : 60 seconds :: 6,160 feet : x feet,**

$$\text{or } x = \frac{6{,}160 \times 60}{5.5} = 67{,}200 \text{ feet. Ans.}$$

(137) We will first reduce 8 hr. 40 min. to minutes. 8 hr. + 40 min. = (8 × 60 min.) + 40 min. = 520 min. In this problem we are required to find "time." We know that a railway train runs 444 miles in 520 minutes, and we want to know how long it will take it to run 1,060 miles at the same rate of speed. It is evident that the time it requires to run 1,060 miles bears the same relation to the time it takes to run 444 miles that 1,060 miles bears to 444 miles. Letting x occupy any place in the proportion, we have the following, the value of x being the same in each. Thus,

(*a*) 1,060 miles : 444 miles :: x min. : 520 min.,

$$\text{or } x = \frac{1{,}060 \times 520}{444} = \frac{551{,}200}{444} =$$

```
 1060                444)551200.00(1241.44+ min.
  520                    444
------                   ----
 21200                   1072
5300                      888
------                   ----
551200                    1840
                          1776
                          ----
                            640
                            444
                            ----
                            1960
                            1776
                            ----
                             1840
                             1776
                             ----
                               64
```

Reducing 1,241.44 min. to hours by dividing by 60, we have

$$\begin{array}{l} 60)1241.44(20 \text{ hr. } 41.44 \text{ min. Ans.} \\ \underline{120} \\ 41 \end{array}$$

(*b*) 444 miles : 1,060 miles :: 520 min. : x min.

$$x = \frac{1,060 \times 520}{444} = 1,241.44 \text{ min., or } 20 \text{ hr. } 41.44 \text{ min. Ans.}$$

(*c*) x min. : 520 min. :: 1,060 miles : 444 miles.

$$x = \frac{1,060 \times 520}{444} = 1,241.44 \text{ min., or } 20 \text{ hr. } 41.44 \text{ min. Ans}$$

(*d*) 520 min. : x min. :: 444 miles : 1,060 miles.

$$x = \frac{1,060 \times 520}{444} = 1,241.44 \text{ min., or } 20 \text{ hr. } 41.44 \text{ min. Ans.}$$

(138) This is an *inverse* ratio since it will take a pump discharging 85 gal. per minute a *longer* time to fill the tank than one discharging 135 gal. per minute. The direct proportion would be

135 gal. : 85 gal. :: 38 min. : x min.

In order to make the proportion inverse, we must invert one of the couplets, and we have

135 gal. : 85 gal. : x min. : 38 min.

$$x = \frac{135 \times 38}{85} = 60\frac{6}{17} \text{ min. Ans.}$$

$$\begin{array}{r} 135 \\ \times\ 38 \\ \hline 1080 \\ 405 \\ \hline 85)5130(60\frac{6}{17} \\ 510 \\ \hline \frac{30}{85} = \frac{6}{17} \end{array}$$

(139) 8 lb. + 8 lb. + 80 lb. = 96 lb., or the number of pounds in the mixture. The relation between the amount of copper which 36 lb. of this mixture will contain, and the amount which 96 lb. contain is the same as 32 : 96; whence, the proportion,

$$32 \text{ lb.} : 96 \text{ lb.} :: x : 8 \text{ lb.},$$

```
96)256(2
   192
   ---
    64   2
    -- = -
    96   3
```

$$\text{or } x = \frac{32 \times 8}{96} = \frac{256}{96} = 2\frac{2}{3} \text{ lb. Ans.}$$

(140) This will be an *inverse* ratio, since the *larger* wheel will turn a less number of times than the smaller wheel in going a certain distance. The direct proportion is

$$12.56 \text{ ft.} : 15.7 \text{ ft.} :: 520 : x.$$

Inverting one of the couplets,

$$15.7 \text{ ft.} : 12.56 \text{ ft.} :: 520 : x,$$

$$\text{or } x = \frac{12.56 \times 520}{15.7} = 416 \text{ times. Ans.}$$

```
           520
         12.56
         -----
          3120
         2600
        1040
        520
       -------
15.7 ) 6531.20 ( 416 times
       628
       ---
        251
        157
        ---
         942
         942
         ---
```

(141) Before forming the proportion, we will combine the three simple ratios into one by reducing them all to the same denomination. A cistern 28 feet long, 12 feet wide, and 10 feet deep contains $28 \times 12 \times 10 = 3,360$ cubic feet.

Again, a cistern 20 ft. long, 17 ft. wide, and 6 ft. deep contains $20 \times 17 \times 6 = 2,040$ cubic feet. What do we wish to find? In this case it is "barrels." We know that a cistern containing 3,360 cubic feet holds 798 barrels of water, and we want to know how many barrels of water a cistern containing 2,040 cubic feet will hold. The number of barrels that a cistern containing 2,040 cubic feet will hold bears the same relation to the number of barrels that a cistern containing 3,360 cubic feet holds as 2,040 cubic feet bears to 3,360 cubic feet. Hence,

$$2,040 \text{ cu. ft.} : 3,360 \text{ cu. ft.} :: x \text{ bbl.} : 798 \text{ bbl.},$$

or $$x = \frac{\overset{85}{\cancel{2,040}} \times \overset{57}{\cancel{798}}}{\underset{\underset{10}{\cancel{140}}}{\cancel{3,360}}} = \frac{85 \times 57}{10} = 484.5 = 484\tfrac{1}{2} \text{ bbl. Ans.}$$

Mensuration and Use of Letters in Formulas.

(QUESTIONS 142–218.)

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(142) Substituting for D, x, B, and i their values,

$$C = \frac{D - x}{B + i} = \frac{120 - 12}{10 + 3.5} = \frac{108}{13.5} = 8. \quad \text{Ans.}$$

A line between two numbers signifies that the one above the line, or numerator, is to be divided by the one below the line, or denominator.

(143) Substituting for A, h, D, and x their values,

$$\frac{A\,h + D}{2x + 6} = \frac{(5 \times 200) + 120}{(2 \times 12) + 6} = \frac{1{,}000 + 120}{24 + 6} = \frac{1{,}120}{30} = 37\tfrac{1}{3}.$$

$$37\tfrac{1}{3} + D = 37\tfrac{1}{3} + 120 = 157\tfrac{1}{3}. \quad \text{Ans.}$$

When there is no sign between the letters, multiplication is understood.

(144) Substituting for B, h, A, x, and i their values,

$$r = \frac{3.246 \times B \times h}{\dfrac{A\,x + h}{A\,i - B}} = \frac{3.246 \times 10 \times 200}{\dfrac{(5 \times 12) + 200}{(5 \times 3.5) - 10}} = \frac{6{,}492}{\dfrac{260}{7.5}} =$$

$$6{,}492 \div \frac{260}{7.5} = 6{,}492 \times \frac{7.5}{260} = 187.269 +. \quad \text{Ans.}$$

(145) Substituting for A, D, i, and B their values,

$$v = \sqrt{\frac{A\,D}{i\,B + 1.5}} = \sqrt{\frac{5 \times 120}{(3.5 \times 10) + 1.5}} = \sqrt{\frac{600}{36.5}} =$$

$$\sqrt{16.4383} = 4.05 +. \quad \text{Ans.}$$

The square root sign extends over both numerator and denominator, thus indicating that the square root of the entire fraction is to be extracted.

(146) Substituting for B, x, h, and A their values,

$$u = \sqrt[3]{\frac{B x}{.00018\,h\,(A^2 - x)}} = \sqrt[3]{\frac{10 \times 12}{.00018 \times 200 \times (5^2 - 12)}} =$$

$$\sqrt[3]{\frac{120}{.036 \times (25 - 12)}} = \sqrt[3]{\frac{120}{.036 \times 13}} = \sqrt[3]{\frac{120}{.468}} =$$

$$\sqrt[3]{256.41} + = 6.35 +. \quad \text{Ans.}$$

(147) Substituting for h, D, and A their values,

$$f = \frac{10(h-D)^2}{\sqrt[3]{D+A}} = \frac{10\,(200-120)^2}{\sqrt[3]{120+5}} = \frac{10 \times 80^2}{\sqrt[3]{125}} = \frac{64{,}000}{5} = 12{,}800. \quad \text{Ans.}$$

(148) Substituting for B, A, and D their values,

$$g = \frac{(B - A)^2 - \sqrt[3]{D + A}}{A^3 - (1 + D)} = \frac{(10 - 5)^2 - \sqrt[3]{120 + 5}}{5^3 - (1 + 120)} =$$

$$\frac{5^2 - \sqrt[3]{125}}{125 - 121} = \frac{25 - 5}{4} = \frac{20}{4} = 5. \quad \text{Ans.}$$

(149) Substituting for A, B, and h their values,

$$k = \sqrt{\frac{A\,B^2}{\sqrt[3]{A\,h}}} = \sqrt{\frac{5 \times 10^2}{\sqrt[3]{5 \times 200}}} = \sqrt{\frac{5 \times 100}{\sqrt[3]{1{,}000}}} = \sqrt{\frac{500}{10}} =$$

$$\sqrt{50} = 7.071 +. \quad \text{Ans.}$$

(150) Substituting for A, h, D, x, and B their values,

$$T = \sqrt{\frac{A^2\left[490 + \frac{(h\,x)^2}{D^2}\right]}{h + \frac{x}{D}\,(A^2 - B)^2}} = \sqrt{\frac{5^2\left[490 + \frac{(200 \times 12)^2}{120^2}\right]}{200 + \frac{12}{120}\,(5^2 - 10)^2}} =$$

$$\sqrt{\frac{25\,(490 + 400)}{200 + (\frac{1}{10} \times 225)}} = \sqrt{\frac{25 \times 890}{200 + 22.5}} = \sqrt{\frac{22{,}250}{222.5}} =$$

$$\sqrt{100} = 10. \quad \text{Ans.}$$

(151) When one straight line meets another straight line, two angles are formed which together equal 180°. Hence, if one of the angles = 152° 3′, the other angle = 180° − 152° 3′, or

$$\begin{array}{rr} & 180^\circ = 179^\circ\ 60' \\ \text{subtracting,} & 152^\circ\ \ 3' \\ \hline & 27^\circ\ 57'. \end{array} \quad \text{Ans.}$$

(152) There are 60 seconds in one minute and 60 minutes in one degree; therefore, $140° = 140 \times 60 = 8{,}400$ minutes; $8{,}400' + 17' = 8{,}417'$; $8{,}417' = 8{,}417 \times 60 = 505{,}020$ seconds, and $505{,}020'' + 10'' = 505{,}030''$. Ans.

(153) See Arts. **359** and **360.**

(154) (*a*) $240 \div 60 = 4$, the number of degrees. Ans.

(*b*) $240 \times 60 = 14{,}400$, the number of seconds. Ans.

(155) See Arts. **355-357.**

(156) See Art. **369.** If the rectangle has the same base and altitude, it would have the same area.

(157) No, since the sum of the three shorter sides is not greater than the fourth side.

(158) Since the area is to be found in square inches, the $2\frac{1}{2}$ feet must be reduced to inches. $2\frac{1}{2}$ ft. = 30 in. Area $= 30 \times 11\frac{1}{2} = 345$ sq. in. Ans.

(159) Since there are 144 sq. in. in 1 sq. ft., the area of the zinc in the last example $= \frac{345}{144} = 2.396$ sq. ft. The weight per square foot $= \frac{5.25}{2.396} = 2.19 +$ lb. Ans.

(160) It will take $1\frac{1}{2}$ boards to reach lengthways of the room. Since the room is 15 feet wide, and each board is 5 *inches* wide, it will take $15 \div \frac{5}{12} = 36$ boards, laid side by side, to extend across the width of the room. Hence, number of boards required $= 36 \times 1\frac{1}{2} = 54$. Ans.

(161) By the rule for the area of a trapezoid, area of the land $= \frac{9+6}{2} \times 16 = 120$ square rods. Since there are 160 square rods in one acre, 120 square rods $= \frac{120}{160} = \frac{3}{4}$ of an acre. Ans.

(162) The total area of the floor of the station $= 55 \times 58$ ft. $= 3{,}190$ sq. ft. $- 25 \times 26$ ft. $= 650$ sq. ft., the area represented by the lower right-hand corner of the

figure. Hence, total area of floor $= 3{,}190 - 650 = 2{,}540$ sq. ft.

From this we have to deduct the following areas:

2 boilers	$= 2 \times 8 \times 19$	$=$	$304.$	sq. ft.
Feed pump	$= 2\frac{1}{2} \times 5$	$=$	12.5	sq. ft.
2 engines	$= 2 \times 4\frac{1}{2} \times 10$	$=$	90	sq. ft.
2 dynamos	$= 2 \times 5\frac{1}{2} \times 6\frac{1}{2}$	$=$	71.5	sq. ft.
Switchboard	$= \dfrac{10 \times 3.5}{12}$	$=$	2.92	sq. ft.
			480.92	sq. ft.

The unoccupied floor space, therefore, $=$

$2{,}540 - 480.92 = 2{,}059.08$ sq. ft. Ans.

(163) The length of the walk, neglecting the corners, that is, the distance $a\,b + b\,c + c\,d + d\,a$, Fig. 1, $= 2 \times 528 + 2 \times 352 = 1{,}760$ ft. The width $= 10$ ft.; area $= 1{,}760 \times 10 = 17{,}600$ sq. ft. The area of the four corners $= 4 \times 10^2 = 400$ sq. ft. Total area of walk $= 17{,}600 + 400 = 18{,}000$ sq. ft. Hence the area in sq. yds. $= \dfrac{18{,}000}{9} = 2{,}000$. Ans.

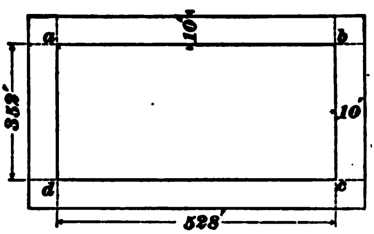

FIG. 1.

(164) The area of the sides of the room $= 2 \times 20 \times 90 = 3{,}600$ sq. ft.; area of ends $= 2 \times 20 \times 50 = 2{,}000$ sq. ft. Total area of walls $= 3{,}600 + 2{,}000 = 5{,}600$ sq. ft.

From this is to be deducted the following areas:

4 doors $= 4 \times 5\frac{1}{2} \times 10 = 220$ sq. ft.

14 windows $= 14 \times 5 \times 11 = 770$ sq. ft.

Length of baseboard, deducting the width of the 4 doors $= 2 \times 90 + 2 \times 50 - 4 \times 5\frac{1}{2} = 180 + 100 - 22 = 258$ feet; width of baseboard $= 9$ inches, or $\frac{3}{4}$ of a foot; area of baseboard $= 258 \times \dfrac{3}{4} = 193.5$ sq. ft. Hence, we have to deduct $220 + 770 + 193.5 = 1{,}183.5$ sq. ft.

Number of sq. ft. of plastering $= 5,600 - 1,183.5 = 4,416.5$ sq. ft.; number of sq. yd. of plastering $= 4,416.5 \div 9 = 490.72$. Ans.

(165) A triangle with three equal angles has three equal sides, and is, therefore, an equilateral triangle.

(166) A triangle with two equal angles has two equal sides, and is, therefore, an isosceles triangle.

(167) No, since the sum of the two shorter sides is not greater than the third side.

(168) (*a*) Draw a line BD from the vertex perpendicular to the base, Fig. 2. It will divide the base into two equal parts, as shown. In the right-angled triangle ABD, the hypotenuse $AB = 6$, and side $AD = 3$; hence, by rule **50**, Art. **385**, BD, the altitude $= \sqrt{6^2 - 3^2} = \sqrt{27} = 5.196$ ft. Ans.

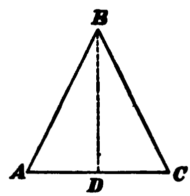

FIG. 2.

(*b*) Area $= \dfrac{6 \times 5.196}{2} = 15.588$ sq. ft. Ans.

(169) The sum of the three angles in any triangle = 2 right angles, or 180°. In the given triangle, the sum of two angles $= 23° + 32° 32' = 55° 32'$, and the third angle $= 180° - 55° 32'$, or

$$\begin{array}{rr} 180° = & 179° \ 60' \\ \text{subtracting,} & 55° \ 32' \\ \hline & 124° \ 28'. \end{array}$$ Ans.

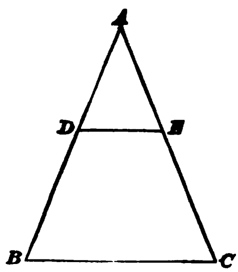

FIG. 3.

(170) In Fig. 3 we have the proportion $AD : DE :: AB : BC$, in which $AD = 10$ in., $AB = 24$ in., and $BC = 13\frac{1}{2}$ in., to find DE.

Substituting the given values,

$$10 : DE :: 24 : 13\tfrac{1}{2}, \text{ or}$$

$$DE = \frac{10 \times 13.5}{24} = 5.625 \text{ in.} \quad \text{Ans.}$$

(171) A line drawn diagonally from one corner to the opposite one would form the hypotenuse of a right-angled

triangle, whose two sides are 39 and 52 feet. By rule **49**, Art. **385**, the length of the diagonal $= \sqrt{52^2 + 39^2} = 65$ ft. Ans.

(172) Using rule **50**, Art. **385**, the required distance $= \sqrt{24^2 - 8^2} = \sqrt{576 - 64} = 22.627$ ft. $= 22$ ft. $7\frac{1}{2}$ in. +. Ans.

(173) See Art. **386**.

(174) (*a*) Using rule **52**, Art. **386**, length of base $=$

$$\frac{2 \times 200}{20} = 20 \text{ in. Ans.}$$

(*b*) A perpendicular let fall from the vertex to the base will divide the triangle into two equal right-angled triangles, of which the perpendicular side is 20 inches and the horizontal side 10 inches. Hence, according to rule **49**, Art. **385**, the hypotenuse $= \sqrt{20^2 + 10^2} = 22.36$ in. $=$ the length of one of the equal sides of the given triangle. Ans.

(175) An equilateral heptagon has seven equal sides; hence, the sum of all the sides, or the perimeter, $= 7 \times 3 = 21$ in. Ans.

(176) A regular decagon has 10 equal sides; hence, the length of one of the sides $= 40 \div 10 = 4$ inches. Ans.

(177) A dodecagon has 12 sides, and by rule **53**, Art. **389**, we have interior angle $\frac{180 \times (12 - 2)}{12} = 150°$. Ans.

(178) Divide the pentagon into five equal isosceles triangles, as shown in Fig. 4, by drawing a line from the center to each angle. The area of one of the triangles, as $A\ B\ C$, $= 43 \div 5 = 8.6$ sq. in. We have given, therefore, the area and base $B\ C$ of the triangle $A\ B\ C$, to find its altitude $A\ D$. Using rule **52**, Art. **386**, $A\ D = \frac{8.6 \times 2}{5} = 3.44$ in., the perpendicular distance from the center to one side. Ans.

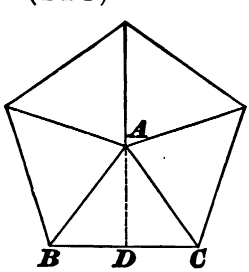

FIG. 4.

(179) See example, Art. **389.** The process is simply to find one of the angles of the polygon, and then to divide it by 2. By rule **53,** Art. **389,** one of the interior angles $= \frac{180 \times (8-2)}{8} = 135°$.

This divided by $2 = 67\frac{1}{2}°$. Ans.

(180) Divide the figure into two triangles and one trapezoid, as shown in Fig. 5. The dimensions of the different parts can then be found by measurement. In obtaining the areas, it will be easier to use decimals than common fractions.

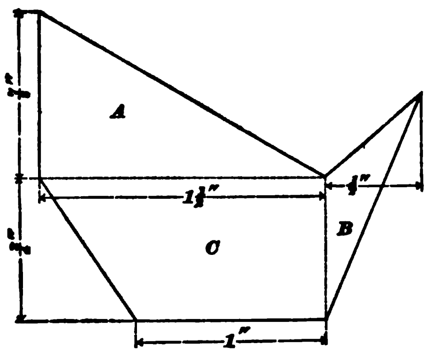

FIG. 5.

Area of triangle $A = \frac{1\frac{1}{2} \times \frac{7}{8}}{2} = \frac{1.5 \times .875}{2} = .656$ sq. in.

Area of triangle $B = \frac{\frac{3}{4} \times \frac{1}{2}}{2} = \frac{.75 \times .5}{2} = .188$ sq. in.

Area of trapezoid $C = \frac{1\frac{1}{2} + 1}{2} \times \frac{3}{4} = \frac{1.5 + 1}{2} \times .75 = 1.25 \times .75 = .938$ sq. in.

Hence, the area of the whole figure $= .656 + .188 + .938 = 1.78 +$ sq. in. Ans.

(181) An angle inscribed in a circle is measured by one-half the intercepted arc. In this case, the angle intercepts one-fourth the circumference, and is measured by one-eighth the circumference, or by $360° \times \frac{1}{8} = 45°$. Hence, there are 45° in the angle. Ans.

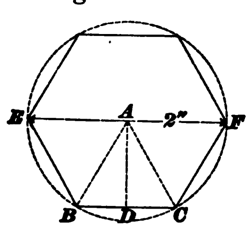

FIG. 6.

(182) Since this is a regular hexagon, it may be inscribed in a circle (Fig. 6), and the radius of the inscribing circle will be equal to one side of the hexagon. Since the diameter $EF = 2$ inches, the radii

$A B$ and $A C$, and the side $B C$ each = 1 inch, and the triangle $A B C$ is equilateral. Draw the line $A D$ perpendicular to the side $B C$; it will bisect $B C$. Then, in the right-angled triangle $A D B$, $A B = 1$, and $B D = \frac{1}{2}''$, to find $A D$. According to rule **50**, Art. **385**, $A D = \sqrt{1^2 - .5^2} = \sqrt{.75} = .866''$. Hence, the distance between two opposite sides of the hexagon $= A D \times 2 = .866 \times 2 = 1.732''$. Ans.

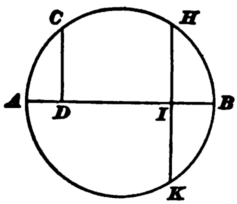

FIG. 7.

(183) In Fig. 7, we have the proportion $BI : HI :: HI : IA$, in which $BI = 6$, and $HI = \frac{1}{2}$ of $HK = \frac{18}{2} = 9$. Substituting, $6 : 9 :: 9 : IA$, or $IA = \frac{81}{6} = 13.5$ in. Hence, the diameter $AB = IA + BI = 13.5 + 6 = 19.5$ in. Ans.

(184) If the diameter $AB = 32\frac{1}{2}$ ft. and $IB = 8$ ft., $AI = 32\frac{1}{2} - 8 = 24\frac{1}{2}$ ft. Then, from the proportion of the last example, $8 : HI :: HI : 24.5$, whence $HI = \sqrt{8 \times 24.5} = \sqrt{196} = 14$ ft., and $HK = 2 \times 14 = 28$ ft. Ans. (See Art. **400.**)

(185) Diameter = circumference $\div$ 3.1416. By rule **56**, Art. **403**, diameter of tree $= 7.854 \div 3.1416 = 2\frac{1}{2}$ ft. Ans.

(186) One mile = 5,280 feet. The circumference of the wheel in feet $= \frac{72 \times 3.1416}{12} = 18.8496$. (See rule **55**, Art. **402.**) Number of revolutions in going one mile $= 5{,}280 \div 18.8496 = 280.112$. Ans.

(187) Using rule **58**, Art. **405**, area = diameter squared $\times .7854$. $6.06^2 = 36.7236$; $36.7236 \times .7854 = 28.8427$ sq. in. Ans.

(188) By rule **59**, Art. **406**, inside diameter $= \sqrt{\frac{113.0976}{.7854}} = 12''$. Since the pipe is $\frac{3}{4}''$ thick, the outside diameter must be $\frac{3}{4}'' \times 2 = 1\frac{1}{2}''$ more, or $12 + 1\frac{1}{2} = 13\frac{1}{2}''$. Ans

(189) Since the radius of the circle = 6 in., its diameter = 12 in., and its circumference = $12 \times 3.1416 = 37.6992$ in. There are 360° in the circumference, and the length of an arc of $12° = 37.6992 \times \frac{12}{360} = 1.25664$ in. Ans.

(190) The area of a circle 22 inches in diameter $= 22^2 \times .7854 = 380.1336$ sq. in. (See rule **58**, Art. **405.**) Area of a circle 21 inches in diameter $= 21^2 \times .7854 = 346.3614$ sq. in. Hence, the area of a flat ring whose outside diameter = 22 in. and inside diameter = 21 in. is $380.1336 - 346.3614 = 33.7722$ sq. in. Ans.

(191) In the formula of rule **61**, Art. **408**, $\frac{4h^2}{3}\sqrt{\frac{D}{h} - .608}$, h = the height of the segment = 5 in., and D = the diameter of the circle = 56 in. Hence, the area of the segment $= \frac{4 \times 5^2}{3}\sqrt{\frac{56}{5} - .608} = \frac{100}{3}\sqrt{10.592} = 33\frac{1}{3} \times 3.255 =$ 108.5 sq. in. Ans.

(192) In Fig. 8, let $A\,C\,B\,D$ represent a section of the largest square bar that can be planed from the round bar. $A\,B$ and $C\,D$ each = 2 in., and in the right-angled triangle $A\,O\,C$, the sides $A\,O$ and $C\,O$ each = 1 in. The hypotenuse $A\,C = \sqrt{1^2 + 1^2} = \sqrt{2} = 1.4142$ in. Ans. Hence, the largest square bar that can be planed from the round bar is 1.4142″ square.

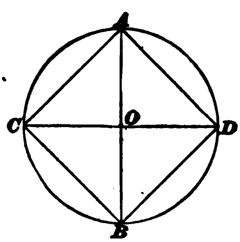

FIG. 8.

(193) The area of a circle 15 in. in diameter $= 15^2 \times .7854 = 176.715$ sq. in. Hence, the area of a sector of this circle, whose angle is $12\frac{1}{2}°$, $= 176.715 \times \frac{12\frac{1}{2}}{360} = \frac{2,208.937}{360} =$ 6.1359 sq. in. Ans. (See rule **60**, Art. **407.**)

(194) (*a*) The side of a square whose area = 103.8691 sq. in. $= \sqrt{103.8691} = 10.1916$ in. Ans.

(*b*) By rule **59,** Art. **406,** the diameter of a circle having the same area $= \sqrt{\frac{103.8691}{.7854}} = 11\frac{1}{2}$ in. Ans.

(*c*) Perimeter of the square $= 10.1916 \times 4 = 40.7664$ in.; circumference of the circle $= 11.5 \times 3.1416 = 36.1284$ in.; difference $= 40.7664 - 36.1284 = 4.638$ in. Ans.

(195) With 5 in. allowed for lap, the length of the plate which forms the shell is $46 - 5 = 41$ in. This is the circumference of the shell. By rule **56,** Art. **403,** the diameter corresponding to this circumference is $\frac{41}{3.1416} =$ 13.05 in. Ans.

(196) The convex area is the area of the outside surface, not including the area of the ends. The circumference of the base $= 26 \times 3.1416 = 81.6816$ in. This reduced to feet, since the area is to be in feet, $= 81.6816 \div 12 = 6.8068$. Using rule **62,** Art. **416,** convex area $= 6.8068 \times 10\frac{1}{2} =$ 71.4714 sq. ft. Ans.

(197) The perimeter of the base $= 4 \times 6 = 24$ in. $= 2$ ft. Convex area $= 2 \times 12 = 24$ sq. ft. The area of the bases is found as follows: In Fig. 9, $AB = 4$ in. and $AC = 2$ in.; since this is a regular hexagon, $AO = AB = 4$ in. By rule **50,** Art. **385,** $OC = \sqrt{4^2 - 2^2} = \sqrt{12} = 3.4641$ in.; area of triangle $AOB = \frac{4 \times 3.4641}{2} = 6.9282$ sq. in.; area of base $= 6.9282 \times 6 = 41.5692$, and the area of both bases $= 41.5692 \times 2 = 83.1384$ sq. in. This reduced to square feet $= \frac{83.1384}{144} = .5774$. Hence, the area of the entire surface of the column is $24 + .5774 = 24.5774$ sq. ft. Ans.

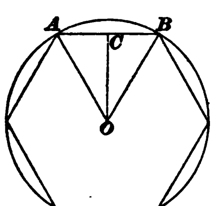

FIG. 9.

(198) The cubical contents in cubic inches = area of base in square inches $\times$ altitude in inches. The area of the base in the last example was found to be 41.5692 sq. in.;

altitude $= 12 \times 12 = 144$ in. Hence, the cubical contents $=$ $41.5692 \times 144 = 5{,}985.9648$ cu. in. Ans.

(199) This example is solved by combining the rules for the circular ring (see example, Art. **406**) and for the cylinder. To obtain the area of one end of the tube, we have $4^2 \times .7854 = 12.5664 =$ area of a circle 4 inches in diameter; $3.73^2 \times .7854 = 10.9272 =$ area of a circle 3.73 inches in diameter; difference $= 12.5664 - 10.9272 = 1.6392 =$ area of one end of the tube. The cubical contents $= 1.6392 \times 12$ $= 19.6704$ cu. in.; the weight $= 19.6704 \times .28 = 5.5$, or $5\frac{1}{2}$ lb. Ans.

(200) This example is done exactly like the one in Art. **417**, and the solution is given here without explanation.

(*a*) In the formula of rule **61**, Art. **408**, $\frac{4h^2}{3}\sqrt{\frac{D}{h} - .608}$, h in this case $= 18$, and $D = 60$.

Substituting, area $=$

$$\frac{4 \times 18^2}{3}\sqrt{\frac{60}{18} - .608} = \frac{4 \times 324}{3}\sqrt{3.333 - .608} = 432 \times \sqrt{2.725} =$$

$432 \times 1.65 = 712.8$ sq. in. This reduced to square feet $= 712.8 \div 144 = 4.95$. Hence, the steam space $= 4.95 \times 16 = 79.2$ cu. ft. Ans.

(*b*) Total area of one end of boiler in square inches $=$ $60^2 \times .7854 = 2{,}827.44$. From this is to be subtracted the area of the tube ends and of the segment found above.

Area of ends of tubes	$= 3.5^2 \times .7854 \times 64 =$	615.75 sq. in.
Area of segment	$=$	712.8 sq. in.
		1,328.55 sq. in.

Area of water space $= 2{,}827.44 - 1{,}328.55 = 1{,}498.89$ sq. in.

Contents of water space $= 1{,}498.89 \times 16 \times 12 = 287{,}786.88$ cu. in., and $287{,}786.88 \div 231 = 1{,}245.83$, number of gallons, or say 1,246 gal. Ans.

(201) (*a*) Area of piston $= 19^2 \times .7854 = 283.529$ sq. in., or 1.9689 square feet (rule **58**, Art. **405**).

Length of stroke plus the clearance $= 1.14 \times 2$ ft. (24 in. $= 2$ ft.) $= 2.28$ ft.

$1.9689 \times 2.28 = 4.489$ cubic feet, or the volume of steam in the small cylinder (rule **63**, Art. **417**).

(*b*) Area of piston $= 31^2 \times .7854 = 754.7694$ sq. in., or 5.2414 square feet.

Length of stroke plus the clearance $= 1.08 \times 2 = 2.16$ ft.

$5.2414 \times 2.16 = 11.321$ cubic feet, or the volume of steam in the large cylinder. Ans.

(*c*) Ratio $= \frac{11.321}{4.489}$ or $2.522 : 1$. Ans.

(202) Let the equilateral triangle $A\,B\,C$, Fig. 10, represent the base of the pyramid. By rule **50**, Art. **385**, the altitude $A\,D$ of the triangle $= \sqrt{10^2 - 5^2} = \sqrt{75} = 8.6602$ in., and according to rule **51**, Art. **386**, the area of the triangle $= \frac{10 \times 8.6602}{2} = 43.301$ sq. in.

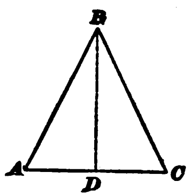

FIG. 10.

Using rule **65**, Art. **423**, volume of pyramid $=$ area of base $\times \frac{1}{3}$ altitude $= \frac{43.301 \times 10}{3} = 144.336$ cu. in. Ans.

(203) In Fig. 11, let $O\,H$ be the altitude and $O\,E$ the slant height of the pyramid. Connect points H and E, forming the right-angled triangle $O\,H\,E$, in which we have to find $O\,H$. Since it is a right pyramid, point H will fall at the center of the base $A\,B\,C\,D$, and, hence, the line $H\,E = \frac{1}{2} A\,B$, or 8 in.; $O\,E = 25$ in.; by rule **50**, Art. **385**, $O\,H = \sqrt{25^2 - 8^2} = \sqrt{561} = 23.6854$ in. Ans.

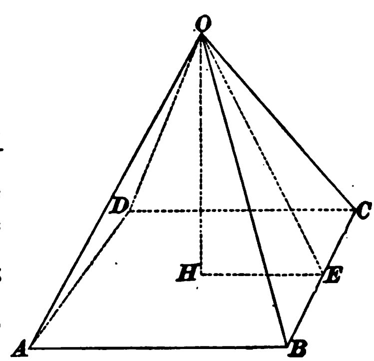

FIG. 11.

(204) The area of the convex surface = circumference of base $\times \frac{1}{2}$ slant height $= 18.8496 \times \frac{10}{2} = 94.248$ sq. in. (See rule **64**, Art. **422.**) The area of the entire surface = 94.248 sq. in. + the area of the base. The diameter of the base $= \frac{18.8496}{3.1416} = 6$ in.; hence, the area of the base $= 6^2 \times .7854 = 28.2744$ (rules **56** and **58**, Arts. **403** and **405**); therefore, the area of the entire surface $= 94.248 + 28.2744 = 122.5224$ sq. in. Ans.

(205) Using rule **65**, Art. **423**, volume = area of base $\times \frac{1}{3}$ altitude $= 28.2744 \times \frac{9}{3} = 84.8232$ cu. in. Ans.

(206) The vat has the form of an inverted frustum of a pyramid. Area of larger base $= 15^2 = 225$ sq. ft.; area of smaller base $= 12^2 = 144$ sq. ft. Hence, by rule **67**, Art. **427**, the contents of the vat in cubic feet $= (225 + 144 + \sqrt{225 \times 144}) \frac{11}{3} = (369 + 180) \times \frac{11}{3} = 549 \times \frac{11}{3} = 2{,}013$ cu. ft. This should be reduced to cubic inches by multiplying by 1,728, the number of cubic inches in a cubic foot. $2{,}013 \times 1{,}728 = 3{,}478{,}464$ cu. in. Since there are 231 cubic inches in a gallon, the number of gallons that the vat will hold $= \frac{3{,}478{,}464}{231} = 15{,}058.29$. Ans.

(207) The pail is in the form of a frustum of a cone. Area of larger base $= 12^2 \times .7854 = 113.0976$ sq. in. Area of smaller base $= 63.6174$ sq. in. Hence, the contents in cubic inches =

$$(113.0976 + 63.6174 + \sqrt{113.0976 \times 63.6174}) \times \frac{11}{3} =$$

$$(176.715 + \sqrt{7{,}194.9753}) \frac{11}{3} = (176.715 + 84.8232) \times \frac{11}{3} =$$

$$261.5382 \times \frac{11}{3} = 958.9734.$$

The contents of the vat in cubic inches were found in the

last example to be 3,478,464. Hence, the number of pails of water required to fill the vat $= 3,478,464 \div 958.9734 = 3,627.28$. Ans.

(208) By rule **66,** Art. **426,** the area of the convex surface = half the sum of the perimeters of the upper and lower bases $\times$ the slant height, or $\frac{48 + 36}{2} \times 32 = 42 \times 32 = 1,344$ sq. in.

(209) See note following the question.

Outside diameter of upper base $= \frac{170.5}{3.1416} =$ about 54.27 in.; inside diameter $= 54.27 - 2.5 = 51.77$ in.; area of upper base $= 51.77^2 \times .7854 = 2,105$ sq. in., nearly.

Outside diameter of lower base $= \frac{190}{3.1416} = 60.48$ in., nearly; inside diameter $= 60.48 - 2.5 = 57.98$ in.; area of lower base $= 57.98^2 \times .7854 = 2,640$ sq. in., nearly.

Apply rule **67,** Art. **427.** Contents of the tank in cubic inches $= (2,105 + 2,640 + \sqrt{2,105 \times 2,640}) \times \frac{7 \times 12}{3} = (4,745 + 2,357) \times 28 = 7,102 \times 28 = 198,856$ cu. in. Hence, the number of gallons $= 198,856 \div 231 = 860.8$. Therefore, in round numbers, the tank will hold 861 gallons.

(210) (*a*) By rule **68,** Art. **429,** area of the surface $= 22.5^2 \times 3.1416 = 506.25 \times 3.1416 = 1,590.435$ sq. in. Ans.

(*b*) Using rule **69,** Art. **430,** the cubical contents = the cube of the diameter $\times .5236 = 11,390.625 \times .5236 = 5,964.1313$ cu. in. Ans.

(211) Having given the area of the surface, to find the volume we must first obtain the diameter. The process is just the reverse of finding the surface when the diameter is given. Hence, the diameter $= \sqrt{201.0624 \div 3.1416} = \sqrt{64} = 8$ in. By rule **69,** Art. **430,** volume $= 8^3 \times .5236 = 268.0832$ cu. in. Ans.

(212) (*a*) Given $OB = \frac{16}{2}$, or 8 inches, and $OA = \frac{13}{2}$, or $6\frac{1}{2}$ inches, to find the volume, area, and weight (see Fig. 12):

Radius of center circle equals $\frac{8 + 6.5}{2}$, or $7\frac{1}{4}$ inches.

Length of center line $= 2 \times 3.1416 \times 7\frac{1}{4} = 45.5532$ inches.

The radius of the inner circle is $6\frac{1}{2}$ inches, and of the outer circle 8 inches; therefore, the diameter of the cross-section on the line AB is $1\frac{1}{2}$ inches. Then, according to rule **70,** Art. **431,** the area of the ring is $1\frac{1}{2} \times 3.1416 \times 45.553 = 214.665$ square inches. Ans.

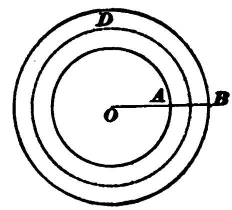

FIG. 12.

Diameter of cross-section of ring $= 1\frac{1}{2}$ inches.

Area of cross-section of ring $= (1\frac{1}{2})^2 \times .7854 = 1.76715$ sq. in. Ans.

By rule **71,** Art. **432,** volume of ring $= 1.76715 \times 45.553 = 80.499$ cu. in. Ans.

(*b*) Weight of ring $= 80.499 \times .261 = 21$ lb. Ans.

(213) The volume of the ring equals the product of its cross-sectional area and its length on line D; therefore, cross-sectional area $= \frac{144.349}{20.42} = 7.069$ sq. in.

The diameter $AB = \sqrt{\frac{7.069}{.7854}} = 3$ in.

Area of surface = circumference of circle, of which AB is diameter, $\times$ length $= 3.1416 \times 3 \times 20.42 = 192.45$ square inches. Ans.

ELEMENTARY ALGEBRA
AND
TRIGONOMETRIC FUNCTIONS.

(QUESTIONS 214–268.)

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(214) See Art. **531.**

(215) (*a*) $3x + 6 - 2x = 7x$. Transposing 6 to the second member, and $7x$ to the first member (Art. **575**),

$$3x - 2x - 7x = -6.$$

Combining like terms, $\quad -6x = -6$;

whence, $\quad x = 1$. Ans.

(*b*) $\quad 5x - (3x - 7) = 4x - (6x - 35)$.

Removing the parentheses (Art. **482**),

$$5x - 3x + 7 = 4x - 6x + 35.$$

Transposing 7 to the second member, and $4x$ and $-6x$ to the first member,

$$5x - 3x - 4x + 6x = 35 - 7. \quad \text{(Art. } \mathbf{575}.)$$

Combining like terms, $4x = 28$;

whence, $\quad x = 28 \div 4 = 7$. Ans.

(*c*) $\quad (x + 5)^2 - (4 - x)^2 = 21x$.

Performing the operations indicated, the equation becomes

$$x^2 + 10x + 25 - 16 + 8x - x^2 = 21x.$$

Transposing, $x^2 - x^2 + 10x + 8x - 21x = 16 - 25$.

Combining like terms, $\quad -3x = -9$.

Dividing by -3, $\quad x = 3$.

(216)

$$
\begin{array}{l}
a^3-a^2-2a-1)2a^6-4a^5-5a^4+3a^3+10a^2+7a+2(2a^3-2a^2-3a-2. \quad \text{Ans.}\\
\qquad 2a^6-2a^5-4a^4-2a^3\\
\hline
\qquad -2a^5-a^4+5a^3+10a^2\\
\qquad -2a^5+2a^4+4a^3+2a^2\\
\hline
\qquad -3a^4+a^3+8a^2+7a\\
\qquad -3a^4+3a^3+6a^2+3a\\
\hline
\qquad -2a^3+2a^2+4a+2\\
\qquad -2a^3+2a^2+4a+2\\
\hline
\end{array}
$$

(217) (*a*)

$$
\begin{array}{l}
2+4a-5a^2-6a^3\\
7a^3\\
\hline
14a^3+28a^4-35a^5-42a^6. \quad \text{Ans. (Art. 493.)}
\end{array}
$$

(*b*)

$$
\begin{array}{l}
4x^2-4y^2+6z^2\\
3x^2y\\
\hline
12x^4y-12x^2y^3+18x^2yz^2. \quad \text{Ans.}
\end{array}
$$

(*c*)

$$
\begin{array}{l}
3b+5c-2d\\
6a\\
\hline
18ab+30ac-12ad. \quad \text{Ans.}
\end{array}
$$

(218) Let $x =$ number of miles he traveled per hour.

Then, $\frac{48}{x} =$ time it took him.

$\frac{48}{x+4} =$ time it would take him if he traveled 4 miles more per hour.

In the latter case the time would have been 6 hours; hence the equation

$$\frac{48}{x+4} = 6.$$

Clearing of fractions, $48 = 6(x+4)$.

Dividing by 6, $8 = x+4$.

Transposing, $8-4 = x$.

$x = 4$. Ans.

(219) The square root of the fraction *a* plus *b* plus *c* over *n*, plus the square root of *a*, plus the fraction *b* plus *c* over *n*, plus the square root of the quantity *a* plus *b*, plus the frac-

tion c over n, plus the parenthesis a plus b, times c, plus a plus bc.

(220) (a) Writing the work as follows, and canceling common factors in both numerator and denominator (Arts. **545** and **546**), we have

$$\frac{9m^2n^2}{8p^3q^3} \times \frac{5p^2q}{2xy} \times \frac{24x^2y^2}{90mn} =$$

$$\frac{9 \times 5 \times 24 \times m^2 \times n^2 \times p^2 \times q \times x^2 \times y^2}{8 \times 2 \times 90 \times m \times n \times p^3 \times q^3 \times x \times y} = \frac{3mnxy}{4pq^2}. \text{ Ans.}$$

(b) This problem may be written as follows, according to Art. **529**,

$$\frac{3ax+4}{1} \times \frac{a^2}{a(3ax+4)\,(3ax+4)}.$$

Canceling a and $(3ax + 4)$, we have $\frac{a}{3ax+4}$. Ans.

(221) See Arts. **602** and **603**.

Sin 17° 28′ = .30015.
Sin 17° 27′ = .29987.

30015 − 29987 = 28, difference for 1′.
28 × $\frac{37}{60}$ = 17, difference for 37″.
29987 + 17 = 30004.

Hence, locating the decimal point,
Sin 17° 27′ 37″ = .30004.

Cos 17° 27′ = .95398.
Cos 17° 28′ = .95389.
95398 − 95389 = 9, difference for 1′.
9 × $\frac{37}{60}$ = 6, difference for 37″.
95398 − 6 = 95392.

Hence, Cos 17° 27′ 37″ = .95392.

Tan 17° 28′ = .31466.
Tan 17° 27′ = .31434.

31466 − 31434 = 32, difference for 1′.
32 × $\frac{37}{60}$ = 20, difference for 37″.
31434 + 20 = 31454.

Hence, Tan 17° 27′ 37″ = .31454.

$$\left.\begin{array}{l}\text{Sin } 17^\circ\ 27'\ 37'' = .30004.\\ \text{Cos } 17^\circ\ 27'\ 37'' = .95392.\\ \text{Tan } 17^\circ\ 27'\ 37'' = .31454.\end{array}\right\}\text{ Ans.}$$

(222) Let $x =$ the capacity.

Then, $x - 42 =$ amount held at first.

$$7(x - 42) = x.$$
$$7x - 294 = x.$$
$$6x = 294.$$
$$x = 49 \text{ gallons.} \quad \text{Ans.}$$

(223) $\dfrac{c(a+b)+cd}{(a+b)c} = \dfrac{ac+bc+cd}{ac+bc}$. Canceling c, which is common to each term, we have $\dfrac{a+b+d}{a+b} = 1 + \dfrac{d}{a+b}$. Ans.

(224) The solution is exactly similar to that of example 221, preceding.

Sin of $63^\circ\ 4'\ 51.8'' = .89165$.
Cos of $63^\circ\ 4'\ 51.8'' = .45274$.
Tan of $63^\circ\ 4'\ 51.8'' = 1.96949$.

(225) $(c^{-\frac{2}{5}})^{-\frac{1}{3}} = c^{\frac{2}{15}}$. Ans.

$$(m\sqrt{n^3})^{-\frac{1}{2}} = m^{-\frac{1}{2}}(n^{\frac{3}{2}})^{-\frac{1}{2}} = m^{-\frac{1}{2}}n^{-\frac{3}{4}} = \frac{1}{m^{\frac{1}{2}}n^{\frac{3}{4}}}. \quad \text{Ans.}$$

$$(cd^{-2})^{\frac{1}{a}} = c^{\frac{1}{a}}d^{-\frac{2}{a}}, \text{ or } \sqrt[a]{cd^{-2}}, \text{ or } \sqrt[a]{\frac{c}{d^2}}. \quad \text{Ans.} \quad (\text{Art. } \mathbf{565.})$$

(226) $\dfrac{3+2x}{2-x} - \dfrac{2-3x}{2+x} + \dfrac{16x-x^2}{x^2-4}$. If the denominator of the third fraction were written $4 - x^2$, instead of $x^2 - 4$, the common denominator would then be $4 - x^2$.

By Art. **531**, $\dfrac{16x-x^2}{x^2-4}$ becomes $-\dfrac{16x-x^2}{-x^2+4} = -\dfrac{16x-x^2}{4-x^2}$.

Hence, $\dfrac{3+2x}{2-x} - \dfrac{2-3x}{2+x} - \dfrac{16x-x^2}{4-x^2}$, when reduced to a common denominator, becomes

$$\frac{(3+2x)\,(2+x)-(2-3x)\,(2-x)-(16x-x^2)}{4-x^2}=$$

$$\frac{(6+7x+2x^2)-(4-8x+3x^2)-(16x-x^2)}{4-x^2}.$$

Removing the parentheses (Art. **482**), we have

$$\frac{6+7x+2x^2-4+8x-3x^2-16x+x^2}{4-x^2}.$$

Combining like terms in the numerator, we have

$$\frac{2-x}{4-x^2}.$$

Factoring the denominator (Art. **523**), we have

$$\frac{2-x}{(2+x)\,(2-x)}.$$

Canceling the common factor $(2-x)$, the result is equal to

$$\frac{1}{2+x},\text{ or }\frac{1}{x+2}.\quad \text{Ans.}\quad (\text{Art. } \mathbf{456}.)$$

(227) $$\frac{a^2+c^2+ac}{a^2+b^2-c^2-2ab}\times\frac{a^2+c^2-b^2-2ac}{a^3c+a^2c^2+ac^3}.$$

Arranging the terms, we have

$$\frac{a^2+ac+c^2}{a^2-2ab+b^2-c^2}\times\frac{a^2-2ac+c^2-b^2}{a^3c+a^2c^2+ac^3},$$

which, by the use of parentheses, becomes

$$\frac{a^2+ac+c^2}{(a^2-2ab+b^2)-c^2}\times\frac{(a^2-2ac+c^2)-b^2}{a^3c+a^2c^2+ac^3}.$$

By Art. **518**, we know that $a^2-2ab+b^2$ and $a^2-2ac+c^2$ are perfect squares, and may be written $(a-b)^2$ and $(a-c)^2$.

Factoring $a^3c+a^2c^2+ac^3$ by Case I, Art. **516** we have

$$\frac{a^2+ac+c^2}{(a-b)^2-c^2}\times\frac{(a-c)^2-b^2}{ac(a^2+ac+c^2)}=$$

$$\frac{a^2+ac+c^2}{(a-b-c)\,(a-b+c)}\times\frac{(a-c-b)\,(a-c+b)}{ac(a^2+ac+c^2)}.$$

(Art. **523.**)

Canceling common factors and multiplying, we have

$$\frac{a-c+b}{(a-b+c)ac}, \text{ or } \frac{a+b-c}{ac(a-b+c)}. \quad \text{Ans.}$$

(228) If none of the terms is similar, the subtraction of one expression from another may be represented only, by connecting the subtrahend with the minuend by means of the sign $-$. Thus, if it is required to subtract $5a^3b - 7a^2b^2 + 5ab^3$ from $a^4 - b^4$, the result will be represented by $a^4 - b^4 - (5a^3b - 7a^2b^2 + 5ab^3)$, which, on removing the parenthesis (Art. **482**), becomes $a^4 - b^4 - 5a^3b + 7a^2b^2 - 5ab^3$. From this result subtract $3a^4 - 4a^3b + 6a^2b^2 + 5ab^3 - 3b^4$.

$$\begin{array}{rl}
a^4 - b^4 - 5a^3b + 7a^2b^2 - 5ab^3 & \textit{minuend.} \\
-3a^4 + 3b^4 + 4a^3b - 6a^2b^2 - 5ab^3 & \textit{subtrahend}, \text{ with signs changed.} \\
\hline
-2a^4 + 2b^4 - a^3b + a^2b^2 - 10ab^3 & \textit{remainder.} \quad \text{(Art. 479.)}
\end{array}$$

Or, $-2a^4 - a^3b + a^2b^2 - 10ab^3 + 2b^4$. Ans. arranged according to the decreasing powers of a.

(229) $.27038 = \sin 15° \ 41' \ 12.9''$. (Art. **606.**)
$.27038 = \cos 74° \ 18' \ 47.1''$.
$2.27038 = \tan 66° \ 13' \ 43.2''$.

(230) (*a*) If the work done *on* a piston by the confined gases be considered positive, then the work done *by* the piston in pushing the gas out of the exhaust-port may be considered negative.

(*b*) While in arithmetic we can add and subtract only positive quantities, in algebra we can perform these operations on both positive and negative quantities.

(231) (*a*) The value of a^0 is 1. (Art. **503.**)

(*b*) $\frac{a^0}{a^{-1}} = a$. Ans. (Art. **565.**)

(232) $-\frac{c-(a-b)}{c+(a+b)} = \frac{(a-b)-c}{c+(a+b)}$. Ans. (Art. **531.**)

(233) (*a*) By Art. **530,** the reciprocal of $\frac{24}{49} = 1 \div \frac{24}{49} = 1 \times \frac{49}{24} = \frac{49}{24}$. Ans.

(*b*) Since, by Art. **530**, a number may be found from its reciprocal by dividing 1 by the reciprocal, the number $= 1 \div 700 = .0014\frac{2}{7}$. Ans.

(234) (*a*)

$$\begin{array}{r} 3a - 2b + 3c \\ \underline{2a - 8b - c} \end{array} \quad \text{becomes} \quad \begin{array}{r} 3a - 2b + 3c \\ \underline{-2a + 8b + c} \\ a + 6b + 4c \end{array}$$

when the signs of the subtrahend are changed. Now, adding each term (with its sign changed) in the subtrahend to its corresponding term in the minuend, we have $(-2a) + (3a) = a$; $(+8b) + (-2b) = +6b$; $(+c) + (3c) = +4c$. Hence, $a + 6b + 4c$ equals the difference. Ans.

(*b*)

$$\begin{array}{l} 2x^3 - 3x^2y + 2xy^3 \\ \underline{x^3 + y^3 - xy^2} \end{array} \text{ becomes } \begin{array}{l} 2x^3 - 3x^2y + 2xy^3 \\ \underline{-x^3 - y^3 + xy^2} \\ x^3 - 3x^2y + 2xy^3 - y^3 + xy^2 \end{array}$$

when the signs of the subtrahend are changed. Adding each term in the subtrahend (with its sign changed) to its corresponding term in the minuend, we have $x^3 - 3x^2y + 2xy^3 - y^3 + xy^2$, which, arranged according to the decreasing powers of x, equals $x^3 - 3x^2y + xy^2 + 2xy^3 - y^3$. Ans.

(*c*)

$$\begin{array}{r} 14a + 4b - 6c - 3d \\ \underline{11a - 2b + 4c - 4d} \end{array}$$

On changing the sign of each term in the subtrahend, the problem becomes

$$\begin{array}{r} 14a + 4b - 6c - 3d \\ \underline{-11a + 2b - 4c + 4d} \\ 3a + 6b - 10c + d \end{array}$$

Adding each term of the subtrahend (with the sign changed) to its corresponding term in the minuend, the difference, or result, is $3a + 6b - 10c + d$. Ans.

(235) The numerical values of the following, when $a = 16$, $b = 10$, and $x = 5$, are:

(*a*) $(ab^2x + 2abx)\,4a = (16 \times 10^2 \times 5 + 2 \times 16 \times 10 \times 5) \times 4 \times 16$. It must be remembered that when no sign is expressed between symbols or quantities, the sign of multiplication is understood.

$(16 \times 100 \times 5 + 2 \times 16 \times 10 \times 5) \times 64 = (8{,}000 + 1{,}600) \times 64 = 9{,}600 \times 64 = 614{,}400$. Ans.

(*b*)
$$2\sqrt{4a} - \frac{2bx}{a-b} + \frac{b-x}{x} =$$
$$2\sqrt{64} - \frac{2 \times 10 \times 5}{16 - 10} + \frac{10-5}{5} =$$
$$16 - \tfrac{100}{6} + 1 = \frac{96 - 100 + 6}{6} = \tfrac{2}{6} = \tfrac{1}{3}. \quad \text{Ans.}$$

(*c*) $(b - \sqrt{a})(x^2 - b^2)(a^2 - b^2) = (10 - \sqrt{16}) \times (5^3 - 10^2) \times (16^2 - 10^2) = (10 - 4)(125 - 100)(256 - 100) = 6 \times 25 \times 156 = 23{,}400$. Ans.

(236) (*a*)
$$\begin{array}{r} 4xyz \\ -3xyz \\ -5xyz \\ 6xyz \\ -9xyz \\ 3xyz \\ \hline -4xyz \end{array} \quad \text{Ans.}$$

The sum of the coefficients of the positive terms we find to be $+13$, since $(+3) + (+6) + (+4) = (+13)$.

When no sign is given before a quantity, the $+$ sign must always be understood. The sum of the coefficients of the negative terms we find to be -17, since $(-9) + (-5) + (-3) = (-17)$. Subtracting the *lesser* sum from the *greater* and prefixing the sign of the greater sum $(-)$ (Art. **471**, Rule II), we have $(+13) + (-17) = -4$. Since the terms are all alike, we have only to annex the common symbols xyz to -4, thereby obtaining $-4xyz$ for the result, or sum.

(*b*)
$$\begin{array}{rrr} 3a^2 + & 2ab + & 4b^2 \\ 5a^2 - & 8ab + & b^2 \\ -a^2 + & 5ab - & b^2 \\ 18a^2 - & 20ab - & 19b^2 \\ 14a^2 - & 3ab + & 20b^2 \\ \hline 39a^2 - & 24ab + & 5b^2. \end{array} \quad \text{Ans.}$$

When adding polynomials, always place like terms under each other. (Art. **478.**)

The coefficient of a^2 in the result will be 39, since $(+14) + (+18) + (-1) +$

$(+5)+(+3)=39$. When the coefficient of a term is not written, 1 is always understood to be its coefficient. (Art. **442.**) The coefficient of ab will be -24, since $(-3)+(-20)+(+5)+(-8)+(+2)=-24$. The coefficient of b^2 will be $(+20)+(-19)+(-1)+(+1)+(+4)=+5$. Hence, the result, or sum, is $39a^2-24ab+5b^2$.

$$
\begin{array}{lllllll}
(c) & 4mn & +3ab & -4c & & & \\
 & +2mn & -4ab & & +3x & +3m^2 & -4p \\
\hline
 & 6mn & -\ ab & -4c & +3x & +3m^2 & -4p. \quad \text{Ans.}
\end{array}
$$

(237) (*a*) See Art. **445.**

(*b*) In multiplication, coefficients are multiplied, and exponents are added. In division, the coefficients of the dividend are divided by those of the divisor, and the exponents of the divisor are subtracted from those of the dividend. See rules of multiplication and division.

(*c*) See Art. **487.**

(238) The side $BC=\sqrt{\overline{AB}^2-\overline{AC}^2}$, or $BC=\sqrt{17.69^2-9.75^2}=\sqrt{217.8736}=14$ ft. 9 in. To find the angle BAC, we have

$$\cos BAC=\frac{AC}{AB}, \text{ or } \cos BAC=\frac{9.75}{17.69}=.55115. \quad \text{(Art. **595.**)}$$

$$.55115=\cos 56^\circ\ 33'\ 15''.$$

Angle $ABC=90^\circ-$ angle BAC, or $90^\circ-56^\circ\ 33'\ 15''=33^\circ\ 26'\ 45''$.

$$\left.\begin{array}{l}\text{Side } BC=14 \text{ ft. } 9 \text{ in.}\\ \text{Angle } BAC=56^\circ\ 33'\ 15''.\\ \text{Angle } ABC=33^\circ\ 26'\ 45''.\end{array}\right\}\text{Ans.}$$

(239) (*a*) $$\frac{9x+20}{36}=\frac{4(x-3)}{5x-4}+\frac{x}{4}.$$

When the denominators contain both simple and compound expressions, it is best to remove the simple expressions first, and then remove each compound expression in order. Then, after each multiplication, the result should be reduced to the simplest form.

Multiplying both sides by 36,

$$9x + 20 = \frac{144(x-3)}{5x-4} + 9x,$$

or
$$\frac{144x - 432}{5x - 4} = 20.$$

Clearing of fractions,

$$144x - 432 = 100x - 80.$$

Transposing and combining,

$$44x = 352;$$

whence, $x = 8$. Ans.

(*b*) $ax - \frac{3a - bx}{2} = \frac{1}{2}$ becomes, when cleared of fractions,

$$2ax - 3a + bx = 1.$$

Transposing and uniting terms,

$$2ax + bx = 3a + 1.$$

Factoring, $(2a + b)x = 3a + 1;$

whence, $x = \frac{3a+1}{2a+b}$. Ans.

(*c*) $am - b - \frac{ax}{b} + \frac{x}{m} = 0$ becomes, when cleared of fractions,

$$abm^2 - b^2m - amx + bx = 0.$$

Transposing, $bx - amx = b^2m - abm^2.$

Factoring, $(b - am)x = bm(b - am);$

whence, $x = \frac{bm(b - am)}{(b - am)} = bm.$ Ans.

(240) (*a*) *a* square *x* square, plus two *a* cube *b* fifth, minus the parenthesis *a* plus *b*.

(*b*) The cube root of *x*, plus *y* times the two-thirds power of the parenthesis *a* minus *n* square.

(*c*) The parenthesis *m* plus *n*, times the square of the parenthesis *m* minus *n*, times the parenthesis *m* minus the fraction *n* over two.

(241) (*a*) $16a^2b^2$; $a^4 + 4ab$; $4a^2 - 16a^3b + 5a^6 + 7ax$.

(*b*) Since the terms are not alike, we can only indicate the sum, connecting the terms by their proper signs. (Art. **476.**)

(*c*) Multiplication: $4ac^2d$ means $4 \times a \times c^2 \times d$. (Art. **441.**)

(242) (*a*) $45x^7y^{10} - 90x^5y^7 - 360x^4y^8 =$
$45x^4y^7(x^3y^3 - 2x - 8y)$. Ans. (Art. **516.**)

(*b*) $a^2b^2 + 2abcd + c^2d^2 = (ab + cd)^2$. Ans. (Art. **519.**)

(*c*) $(a+b)^2 - (c-d)^2 = (a+b+c-d)(a+b-c+d)$. Ans. (Art. **523.**)

(243) (*a*) On removing the vinculum, we have

$$2a - \{3b + [4c - 4a - (2a + 2b)] + [3a - b - c]\}.$$

(Art. **482.**)

Removing the parenthesis,

$$2a - \{3b + [4c - 4a - 2a - 2b] + [3a - b - c]\}.$$

Removing the brackets,

$$2a - \{3b + 4c - 4a - 2a - 2b + 3a - b - c\}.$$

Removing the brace,

$$2a - 3b - 4c + 4a + 2a + 2b - 3a + b + c.$$

Combining like terms, the result is $5a - 3c$. Ans.

(*b*) Removing the parenthesis, we have

$$7a - \{3a - [2a - 5a + 4a]\}.$$

Removing the brackets,

$$7a - \{3a - 2a + 5a - 4a\}.$$

Removing the brace,

$$7a - 3a + 2a - 5a + 4a.$$

Combining terms, the result is $5a$. Ans.

(*c*) Removing the parenthesis, we have

$$a - \{2b + [3c - 3a - a - b] + [2a - b - c]\}.$$

Removing the brackets,

$$a - \{2b + 3c - 3a - a - b + 2a - b - c\}.$$

Removing the brace,

$$a - 2b - 3c + 3a + a + b - 2a + b + c.$$

Combining like terms, the result is $3a - 2c$. Ans.

(244) The hypotenuse $AB = \sqrt{\overline{AC}^2 + \overline{BC}^2}$, or

$AB = \sqrt{17.5^2 + 21.3^2} = \sqrt{759.94} = 27.57$, nearly. Ans.

$$\text{Tan } B = \frac{AC}{BC} = \frac{17.5}{21.3} = .82160. \quad (\text{Art. } \mathbf{596.})$$

$$.82160 = \tan 39° \, 24' \, 23''.$$

Angle $B = 39° \, 24' \, 23''$. Ans.

Angle $A = 90° -$ angle $B = 90° - 39° \, 24' \, 23'' = 50° \, 35' \, 37''$. Ans.

(245)
$$t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2}{W_1 s_1 + W_2 s_2}.$$

In order to transform this formula so that t_2 may stand alone in the first member, we must first clear of fractions. Performing this operation, we have

$$t W_1 s_1 + t W_2 s_2 = W_1 s_1 t_1 + W_2 s_2 t_2.$$

Transposing, we have

$$- W_2 s_2 t_2 = W_1 s_1 t_1 - t W_1 s_1 - t W_2 s_2.$$

Factoring, we have

$$- W_2 s_2 t_2 = W_1 s_1 t_1 - (W_1 s_1 + W_2 s_2) t;$$

whence, $$t_2 = \frac{(W_1 s_1 + W_2 s_2)\, t - W_1 s_1 t_1}{W_2 s_2}. \quad \text{Ans.}$$

(246) Let $x =$ the part of the work which they all can do in 1 day when working together.

Then, since $\frac{1}{7\frac{1}{2}} = \frac{2}{15}$, $\quad \frac{1}{5} + \frac{1}{6} + \frac{2}{15} = x;$

or, clearing of fractions and adding,

$$15 = 30x, \text{ and } x = \tfrac{1}{2}.$$

Since they can do $\frac{1}{2}$ the work in 1 day, they can do all the work in 2 days. Ans.

(247) (*a*)
$$\frac{\dfrac{1}{1-x} - \dfrac{1}{1+x}}{\dfrac{1}{1-x} + \dfrac{1}{1+x}} = \frac{\dfrac{1+x-1+x}{1-x^2}}{\dfrac{1+x+1-x}{1-x^2}} =$$

$$\frac{2x}{1-x^2} \div \frac{2}{1-x^2} = \frac{2x}{1-x^2} \times \frac{1-x^2}{2} = x. \quad \text{Ans. (See Art. } \mathbf{559.})$$

(*b*) $$\frac{\frac{a^2}{b^3}+\frac{1}{a}}{\frac{a}{b^2}-\frac{a-b}{ab}}=\frac{\frac{a^3+b^3}{ab^3}}{\frac{a^2b-b^2(a-b)}{ab^3}}=\frac{\frac{a^3+b^3}{ab^3}}{\frac{a^2b-ab^2+b^3}{ab^3}}=$$

$$\frac{a^3+b^3}{ab^3}\div\frac{a^2b-ab^2+b^3}{ab^3}=\frac{a^3+b^3}{ab^3}\times\frac{ab^3}{b(a^2-ab+b^2)}=$$

$$\frac{a+b}{b}.\quad\text{Ans.}$$

(*c*) $$\frac{1}{x+\frac{1}{1+\frac{x+1}{3-x}}}=\frac{1}{x+\frac{3-x}{4}}=\frac{4}{3x+3}.\quad\text{Ans.}$$

(Art. **559.**)

(248) (*a*) $6a^4b^4+a^3b^2-7a^2b^3+2abc+3$.

(*b*) $3+2abc+a^3b^2-7a^2b^3+6a^4b^4$.

(*c*) $1+ax+a^2+2a^3$. Written like this, the a in the second term is understood as having 1 for an exponent; hence, if we represent the first term by a^0, in value it will be equal to 1, since $a^0=1$. Therefore, 1 should be written as the first term when arranged according to the increasing powers of a.

(249) (*a*) According to Art. **563,** $x^{\frac{3}{4}}$ expressed radically is $\sqrt[4]{x^3}$;

$$3x^{\frac{1}{2}}y^{-\frac{3}{2}}\text{ expressed radically is }3\sqrt{xy^{-3}};$$

$$3x^{\frac{1}{6}}y^{-\frac{5}{6}}z^{\frac{1}{3}}=3\sqrt[6]{xy^{-5}z^2},\text{ since }z^{\frac{1}{3}}=z^{\frac{2}{6}}.\quad\text{Ans.}$$

(*b*) (See Art. **565.**) $a^{-1}b^{\frac{1}{2}}+\dfrac{c^{-2}}{a+b}+(m-n)^{-1}-\dfrac{a^2b^{-2}c}{c^{-3}}=$

$$\frac{b^{\frac{1}{2}}}{a}+\frac{1}{c^2(a+b)}+\frac{1}{m-n}-\frac{a^2c^4}{b^2}.\quad\text{Ans.}$$

(*c*) $\sqrt[7]{x^6}=x^{\frac{6}{7}}$. Ans. $\sqrt[8]{x^{-4}}=x^{-\frac{4}{8}}$. Ans.

$$(\sqrt[4]{b^5x^2})^3=(b^{\frac{5}{4}}x^{\frac{1}{2}})^3=b^{\frac{15}{4}}x^{\frac{3}{2}}.\quad\text{Ans.}$$

(250) (*a*) $$\frac{2ax+x^2}{a^3-x^3}\div\frac{x}{a-x}=\frac{x(2a+x)}{a^3-x^3}\times\frac{a-x}{x}.$$

(Art. **549.**)

Canceling common factors, the result is $\frac{2a+x}{a^2+ax+x^2}$. Ans.

$$\begin{array}{l} a-x\,)\,a^3-x^3\,(\,a^2+ax+x^2 \\ \qquad\quad \underline{a^3-a^2x} \\ \qquad\qquad\quad a^2x-x^3 \\ \qquad\qquad\quad \underline{a^2x-ax^2} \\ \qquad\qquad\qquad\quad ax^2-x^3 \\ \qquad\qquad\qquad\quad \underline{ax^2-x^3} \end{array}$$

(*b*) Inverting the divisor and factoring, we have

$$\frac{3n(2m^2n-1)}{(2m^2n-1)(2m^2n-1)}\times\frac{(2m^2n+1)(2m^2n-1)}{3n}.$$

Canceling common factors, the result is $2m^2n+1$. Ans.

(*c*) $9+\frac{5y^2}{x^2-y^2}\div\left(3+\frac{5y}{x-y}\right)$ simplified gives $\frac{9x^2-4y^2}{x^2-y^2}\div\frac{3x+2y}{x-y}$.

Inverting the divisor, we have $\frac{9x^2-4y^2}{x^2-y^2}\times\frac{x-y}{3x+2y}$. Canceling common factors, the result is $\frac{3x-2y}{x+y}$. Ans.

(251) (*a*)
$$\begin{array}{l} 2x^3+2x^2+2x-2 \\ \underline{\quad x-1\qquad\qquad\qquad} \\ 2x^4+2x^3+2x^2-2x \\ \underline{\qquad -2x^3-2x^2-2x+2} \\ 2x^4 \qquad\qquad\qquad -4x+2. \end{array}$$
Ans.

(*b*)
$$\begin{array}{l} x^2-4ax+c \\ \underline{2x+a\qquad\qquad\quad} \\ 2x^3-8ax^2+2cx \\ \underline{\qquad\quad ax^2 \qquad -4a^2x+ac} \\ 2x^3-7ax^2+2cx-4a^2x+ac. \end{array}$$
Ans.

(*c*)
$$\begin{array}{l} -a^3+3a^2b-2b^3 \\ \underline{\quad 5a^3+9ab\qquad\qquad} \\ -5a^6+15a^4b-10a^2b^3 \\ \underline{\qquad -9a^4b \qquad\qquad +27a^3b^2-18ab^4} \\ -5a^6+6a^4b-10a^2b^3+27a^3b^2-18ab^4 \end{array}$$

Arranging the terms according to the decreasing powers of a, we have

$$-5a^5 + 6a^4b + 27a^3b^2 - 10a^2b^3 - 18ab^4. \quad \text{Ans.}$$

(252) (a) $\dfrac{x}{x-y} + \dfrac{x-y}{y-x}$. If the denominator of the second fraction were written $x-y$, instead of $y-x$, then $x-y$ would be the common denominator.

By Art. **531**, the signs of the denominator and the sign before the fraction $\dfrac{x-y}{y-x}$ may be changed, giving $-\dfrac{x-y}{x-y}$. We now have

$$\frac{x}{x-y} - \frac{x-y}{x-y} = \frac{x-x+y}{x-y} = \frac{y}{x-y}. \quad \text{Ans.}$$

(b) $\dfrac{x^2}{x^2-1} + \dfrac{x}{x+1} - \dfrac{x}{1-x}$. If we write the denominator of the third fraction $x-1$ instead of $1-x$, x^2-1 will then be the common denominator.

By Art. **531**, the signs of the denominator and the sign before the fraction may be changed, thereby giving $\dfrac{x}{x-1}$. We now have

$$\frac{x^2}{x^2-1} + \frac{x}{x+1} + \frac{x}{x-1} = \frac{x^2 + x(x-1) + x(x+1)}{x^2-1} =$$

$$\frac{x^2 + x^2 - x + x^2 + x}{x^2-1} = \frac{3x^2}{x^2-1}. \quad \text{Ans.}$$

(c) $$\frac{3a-4b}{7} - \frac{2a-b+c}{3} + \frac{13a-4c}{12}$$

when reduced to a common denominator =

$$\frac{12(3a-4b) - 28(2a-b+c) + 7(13a-4c)}{84}.$$

Expanding the terms and removing the parentheses, we have

$$\frac{36a - 48b - 56a + 28b - 28c + 91a - 28c}{84}.$$

Combining like terms in the numerator, we have as the result,

$$\frac{71a - 20b - 56c}{84}. \quad \text{Ans.}$$

(253) (*a*) Factoring each expression (Art. **519**), we have

$9x^4 + 12x^2y^2 + 4y^4 = (3x^2 + 2y^2)(3x^2 + 2y^2) = (3x^2 + 2y^2)^2$. Ans.

(*b*) $49a^4 - 154a^2b^2 + 121b^4 = (7a^2 - 11b^2)(7a^2 - 11b^2) = (7a^2 - 11b^2)^2$. Ans.

(*c*) $64x^2y^2 + 64xy + 16 = 16(2xy + 1)^2$. Ans.

(254) (*a*) Arrange the dividend according to the decreasing powers of x and divide. Thus,

$$
\begin{array}{l}
3x - 1\,)\,9x^3 + 3x^2 + x - 1\,(\,3x^2 + 2x + 1. \quad \text{Ans.} \\
\qquad\quad \underline{9x^3 - 3x^2} \\
\qquad\qquad\quad 6x^2 + x \\
\qquad\qquad\quad \underline{6x^2 - 2x} \\
\qquad\qquad\qquad\quad 3x - 1 \\
\qquad\qquad\qquad\quad \underline{3x - 1}
\end{array}
$$

(*b*)

$$
\begin{array}{l}
a - b\,)\,a^3 - 2ab^2 + b^3\,(\,a^2 + ab - b^2. \quad \text{Ans.} \\
\qquad\quad \underline{a^3 - a^2b} \\
\qquad\qquad\quad a^2b - 2ab^2 \\
\qquad\qquad\quad \underline{a^2b - ab^2} \\
\qquad\qquad\qquad\quad - ab^2 + b^3 \\
\qquad\qquad\qquad\quad \underline{- ab^2 + b^3}
\end{array}
$$

(*c*) Arranging the terms of the dividend according to the decreasing powers of x, we have

$$
\begin{array}{l}
7x - 3\,)\,7x^3 - 24x^2 + 58x - 21\,(\,x^2 - 3x + 7. \quad \text{Ans.} \\
\qquad\quad \underline{7x^3 - 3x^2} \\
\qquad\qquad - 21x^2 + 58x \\
\qquad\qquad \underline{- 21x^2 + 9x} \\
\qquad\qquad\qquad\quad 49x - 21 \\
\qquad\qquad\qquad\quad \underline{49x - 21}
\end{array}
$$

(255) See Arts. **435** and **436.**

(256) (*a*)

$$1+2x-\frac{4x-4}{5x}=\frac{5x+10x^2-4x+4}{5x}=\frac{10x^2+x+4}{5x}.\text{ Ans.}$$

(Art. **555.**)

(*b*) $\dfrac{3x^2+2x+1}{x+4}=3x-10+\dfrac{41}{x+4}$. Ans. (Art. **554.**)

$$\begin{array}{rl} x+4\,)\,3x^2+\ \ 2x+1 & (\,3x-10+\dfrac{41}{x+4} \\ 3x^2+12x & \\ \hline -10x+\ \ 1 & \\ -10x-40 & \\ \hline 41 & \end{array}$$

(257) The angle $B = 90° -$ angle $A = 90° - 65° 13' 29'' = 24° 46' 31''$. Ans.

The side $BC = AB \times \sin A = 5.5$ yd. $\times \sin 65° 13' 29'' = 5.5$ yd. $\times .90796 = 4.9938$ yd. (Art. **609.**) 4.9938 yd. = 14 ft. $11\frac{3}{4}$ in. Ans.

Side $AC = AB \times \cos A = 5.5$ yd. $\times .41906 = 2.3048$ yd. = 6 ft. 11 in., nearly. Ans.

(258) (*a*) $(x^4-1) \div (x^2+1) = (x^2-1)(x^2+1) \div (x^2+1) = x^2-1$ Ans. (Art. **523.**)

(*b*) $x^4-2x^2y^2+y^4=(x^2-y^2)^2$. (Art. **519.**) $(x^2-y^2) = (x+y)(x-y)$. (Art. **523.**)

Then $(x^2-y^2)^2 = (x^2-y^2)(x+y)(x-y)$. Dividing this latter quantity by $(x-y)$ we have $(x^2-y^2)(x+y)$. Ans.

Note that $x-y$ is a factor of $(x^2-y^2)^2$ and hence of $x^4-2x^2y^2+y^4$.

(259) (*a*) $\dfrac{10x+3}{3}-\dfrac{6x-7}{2}=10(x-1)$.

Reducing the last member to a simpler form, the equation becomes

$$\frac{10x+3}{3}-\frac{6x-7}{2}=10x-10.$$

Clearing of fractions by multiplying each term of both members by 6, the common denominator, and changing the

sign of each term of the numerator of the second fraction, since it is preceded by the minus sign, we have

$$20x + 6 - 18x + 21 = 60x - 60.$$

Transposing terms, $20x - 18x - 60x = -60 - 21 - 6.$
Combining like terms, $-58x = -87.$
Changing signs, $58x = 87;$
hence, $x = \frac{87}{58} = 1\frac{1}{2}.$ Ans.

(*b*) $$(a^2 + x)^2 = x^2 + 4a^2 + a^4.$$

Performing the operation indicated in the first member, the equation becomes

$$a^4 + 2a^2x + x^2 = x^2 + 4a^2 + a^4.$$

Canceling a^4 and x^2 (Art. **576**),

$$2a^2x = 4a^2.$$

Dividing by $2a^2$, $x = 2.$ Ans.

(*c*) $$\frac{x-1}{x-2} - \frac{x+1}{x+2} = \frac{3}{x^2-4}.$$

Clearing of fractions, the equation becomes

$$(x-1)(x+2) - (x+1)(x-2) = 3.$$

Expanding, $x^2 + x - 2 - x^2 + x + 2 = 3.$
Uniting terms, $2x = 3.$
$x = \frac{3}{2} = 1\frac{1}{2}.$ Ans.

(260)

$$\begin{array}{r} 159^\circ\ 27'\ 34.6'' \\ 25^\circ\ 16'\ \ 8.7'' \\ 3^\circ\ 48'\ 53.\ '' \\ \hline 188^\circ\ 32'\ 36.3'' \end{array}$$

(261) (*a*) $x + y + z - (x - y) - (y + z) - (-y)$ becomes $x + y + z - x + y - y - z + y$ on the removal of the parentheses. (Art. **482.**)

Combining like terms, $x - x + y + y - y + y + z - z = 2y.$ Ans.

(*b*) $(2x - y + 4z) + (-x - y - 4z) - (3x - 2y - z)$ becomes $2x - y + 4z - x - y - 4z - 3x + 2y + z$ on the

removal of the parentheses. (Arts. **482** and **483.**) Combining like terms,

$$2x - x - 3x - y - y + 2y + 4z - 4z + z = z - 2x. \quad \text{Ans.}$$

(*c*) $a - [2a + (3a - 4a)] - 5a - \{6a - [(7a + 8a) - 9a]\}.$

In this expression we find aggregation marks of different shapes, thus, [, (, and {. In such cases look for the corresponding part (whatever may intervene), and all that is included between the two parts of each aggregation mark must be treated as directed by the sign before it (Arts. **482** and **483**), no attention being given to any of the other aggregation marks. It is always best to begin with the *innermost* pair, and remove each pair of aggregation marks in order. First removing the parentheses, we have

$$a - [2a + 3a - 4a] - 5a - \{6a - [7a + 8a - 9a]\}.$$

Removing the brackets, we have

$$a - 2a - 3a + 4a - 5a - \{6a - 7a - 8a + 9a\}.$$

Removing the brace, we have

$$a - 2a - 3a + 4a - 5a - 6a + 7a + 8a - 9a.$$

Combining like terms, the result is $-5a$. Ans.

(262) (*a*) $-7my\,\overline{)\,35m^3y + 28m^2y^2 - 14my^3}$

$-5m^2 - 4my + 2y^2$. Ans.

(Art. **506.**)

(*b*) $a^4\,\overline{)\,4a^4 - 3a^5b - a^6b^2}$

$4 - 3ab - a^2b^2$. Ans.

(*c*) $4x^2\,\overline{)\,4x^3 - 8x^5 + 12x^7 - 16x^9}$

$x - 2x^3 + 3x^5 - 4x^7$. Ans.

(263) Let $x =$ the length of the post.

Then, $\frac{x}{5} =$ the part in the earth.

$\frac{3x}{7} =$ the part in the water.

From the conditions of the problem, we have therefore the following statement:

$$\frac{x}{5}+\frac{3x}{7}+13=x;$$

from which $\quad 7x+15x+455=35x;$

$$-13x=-455;$$

and $\quad x=35$ feet. Ans.

(264) Let $\quad x=$ the whole quantity.

Then, $\quad \frac{2x}{3}+10=$ the quantity of niter.

$\frac{x}{6}-4\frac{1}{2}=$ the quantity of sulphur.

$\frac{1}{7}\left(\frac{2x}{3}+10\right)-2=$ the quantity of charcoal.

Hence, $\quad x=\frac{2x}{3}+10+\frac{x}{6}-4\frac{1}{2}+\frac{1}{7}\left(\frac{2x}{3}+10\right)-2.$

Clearing of fractions and expanding terms,

$$42x=28x+420+7x-189+4x+60-84.$$

Transposing,

$$42x-28x-7x-4x=420-189+60-84.$$

$$3x=207.$$

$x=69$ lb., the quantity of gunpowder. Ans.

$\frac{2x}{3}+10=\frac{2\times 69}{3}+10=56$ lb., the quantity of niter. Ans.

$\frac{x}{6}-4\frac{1}{2}=\frac{69}{6}-4\frac{1}{2}=7$ lb., the quantity of sulphur. Ans.

56 lb. $\times\ \frac{1}{7}-2$ lb. $=6$ lb., the quantity of charcoal. Ans.

(265) (*a*) See Art. **568.** The cube root of -125 is -5. Dividing each of the exponents of the literal part by 3, the index of the root, the cube root of $x^3y^6z^9$ is $x^{\frac{3}{3}}y^{\frac{6}{3}}z^{\frac{9}{3}}=xy^2z^3$; hence, $\quad \sqrt[3]{-125x^3y^6z^9}=-5xy^2z^3$. Ans.

(*b*) and (*c*) Proceed exactly as in (*a*). $\sqrt[4]{10,000}=\pm 10$ and $\sqrt[5]{243}=3$; $\sqrt[4]{a^{16}b^{20}c^8}=a^{\frac{16}{4}}b^{\frac{20}{4}}c^{\frac{8}{4}}=a^4b^5c^2$, and $\sqrt[5]{m^{15}n^{20}}=m^3n^4$.

Hence, $\quad \sqrt[4]{10,000a^{16}b^{20}c^8}=\pm 10a^4b^5c^2$. Ans.

$$\sqrt[5]{243m^{15}n^{20}} = 3m^3n^4. \quad \text{Ans.}$$

(*d*) Dividing the exponent of each letter in both numerator and denominator by 5, the index of the root,

$$\sqrt[5]{-\frac{x^5y^{10}z^{15}}{a^{20}b^{15}c^{10}d^5}} = -\frac{xy^2z^3}{a^4b^3c^2d}. \quad \text{Ans.}$$

(266) Using the proportion of Art. **615,**

$$A\,B : B\,C = \sin C : \sin A,$$

or $$70 : \ 42 = \sin C : \sin 36° \ 10'.$$

Hence, $\sin C = \frac{70}{42} \times \sin 36° \ 10' = \frac{70}{42} \times .59014 = .98357.$

The angle whose sine is .98357 is 79° 36′; hence angle $C =$ 79° 36′. Ans.

Angle $B = 180° - (A + C) = 180° - (36° \ 10' + 79° \ 36') =$ 64° 14′. Ans.

Using the proportion again,

$$A\,C : B\,C = \sin B : \sin A;$$

or $A\,C$: 42 ft. = sin 64° 14′ : sin 36° 10′ = .90057 : .59014.

Hence, $A\,C = \dfrac{42 \text{ ft.} \times .90057}{.59014} = 64.1$ ft. nearly. Ans.

(267) (*a*) See Art. **591.**

$$180° - 72° \ 11' \ 36'' = 107° \ 48' \ 24''. \quad \text{Ans.}$$

(*b*) See Art. **590.**

$$90° - 22° \ 34' \ 17'' = 67° \ 25' \ 43''. \quad \text{Ans.}$$

(268) Angle $B = 180° - (A + C) =$

$$180° - (57° \ 34.5' + 44° \ 22.5') = 78° \ 3'. \quad \text{Ans.}$$

$$A\,C : A\,B = \sin B : \sin C = \sin 78° \ 3' : \sin 44° \ 22.5'$$
$$= .97833 : .69936.$$

Hence, $A\,C = \dfrac{344 \text{ ft.} \times .97833}{.69936} = 481.22$ ft. Ans.

$$B\,C : A\,B = \sin A : \sin C = \sin 57° \ 34.5' : \sin 44° \ 22.5'$$
$$= .8441 : .69936.$$

Hence, $B\,C = \dfrac{344 \text{ ft.} \times .8441}{.69936} = 415.19$ ft. Ans.

ELEMENTARY MECHANICS.

(QUESTIONS 355–453.)

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(355) Use formulas **18** and **8**.

Time it would take the ball to fall to the ground $= t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5.5}{32.16}} = .58484$ sec.

The space passed through by a body having a velocity of 500 ft. per sec. in .58484 of a second $= S = Vt = 500 \times .58484 = 292.42$ ft. Ans.

(356) Use formula **7**.

$$\frac{\frac{80}{12} \times 3.1416 \times 160}{60} = 55.85 \text{ ft. per sec. Ans.}$$

(357) $160 \div 60 \times 7 = \frac{8}{21}$ revolution in $\frac{1}{7}$ sec. $360° \times \frac{8}{21} = 137\frac{1}{7}° = 137° \, 8' \, 34\frac{2}{7}''$. Ans.

(358) (*a*) See Fig. 20. $36'' = 3'$. $4 \div 3 = \frac{4}{3} =$ number of revolutions of pulley to one revolution of fly-wheel. $54 \times \frac{4}{3} = 72$ revolutions of pulley and drum per min. $100 \div \left(\frac{18}{12} \times 3.1416\right) = 21.22$ revolutions of drum to raise elevator 100 ft. $\frac{21.22}{72} \times 60 = 17.68$ sec. to travel 100 ft. Ans.

(*b*) $21.22 : x :: 30 : 60$, or $x = \frac{21.22 \times 60}{30} = 42.44$ rev.

per min. of drum. The diameter of the pulley divided by

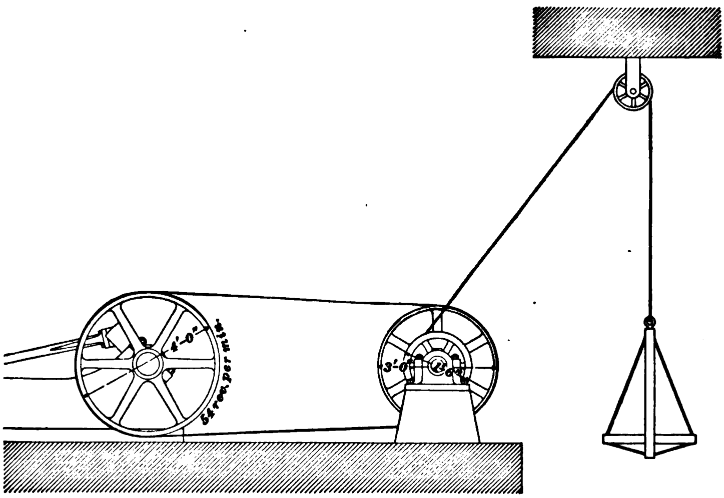

FIG. 20.

the diameter of the fly-wheel $= \frac{3}{4}$, which multiplied by $42.44 = 31.83$ revolutions per min. of fly-wheel. Ans.

(359) See Arts. **857** and **859.**

(360) See Art. **861.**

(361) See Arts. **843** and **871.**

(362) See Art. **871.**

(363) See Arts. **842, 886, 887,** etc.

The relative weight of a body is found by comparing it with a given standard by means of the balance. The absolute weight is found by noting the pull which the body will exert on a spring balance.

The absolute weight increases and decreases according to the laws of weight given in Art. **890;** the relative weight is always the same.

(364) See Art. **861.**

(365) See Art. **857.**

(366) See Art. **857.**

(367) If the mountain is at the same height above, and the valley at the same depth below sea-level respectively, it will weigh more at the bottom of the valley.

(368) $\frac{31,680}{5,280} = 6$ miles. Using formula **12,** $d^2 : R^2 :: W : w$, we have $w = \frac{R^2 W}{d^2} = \frac{3,960^2 \times 20,000}{3,966^2} = 19,939.53 +$ lb. $= 19,939$ lb. $8\frac{1}{2}$ oz. Ans.

(369) Using formula **11,** $R : d :: W : w$, we have $w = \frac{d\,W}{R} = \frac{3,958 \times 20,000}{3,960} = 19,989.89$ lb. $= 19,989$ lb. $14\frac{1}{4}$ oz. Ans.

(370) See Art. **870.**

(371) See Art. **894.**

(372) The velocity which a body may have at the instant the time begins to be reckoned.

(373) Because the man after jumping tends to continue in motion with the same velocity as the train, and the sudden stoppage by the earth causes a shock, the severity of which varies with the velocity of the train.

(374) See Arts. **870** and **871.**

(375) See Art. **872.**

(376) That force which will produce the same final effect upon a body as all the other forces acting together is called the resultant.

(377) (*a*) If a 5-in. line = 20 lb., a 1-in. line = 4 lb. $1 \div 4 = \frac{1}{4}$ in. = 1 lb. Ans. (*b*) $6\frac{1}{4} \div 4 = 1.5625$ in. = $6\frac{1}{4}$ lb. Ans.

(378) Those forces by which a given force may be

replaced, and which will produce the same effect upon a body.

(379) Southeast, in the direction of the diagonal of a square. See Fig. 21.

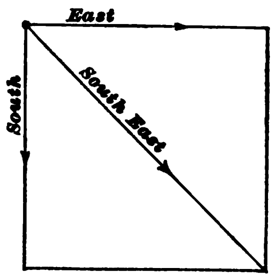

Fig. 21.

(380) $4' \, 6'' = 54''$. $54 \times 2 \times \frac{3}{4} \times .261 = 21.141$ lb. = weight of lever. Center of gravity of lever is in the middle, at a, Fig. 22, 27″ from each end. Consider that the lever has no weight. The center of gravity of the two weights is at b, at a distance from c equal to $\frac{47 \times 54}{47 + 71} = 21.508'' = bc$. Formula **20**, Art. **911**.

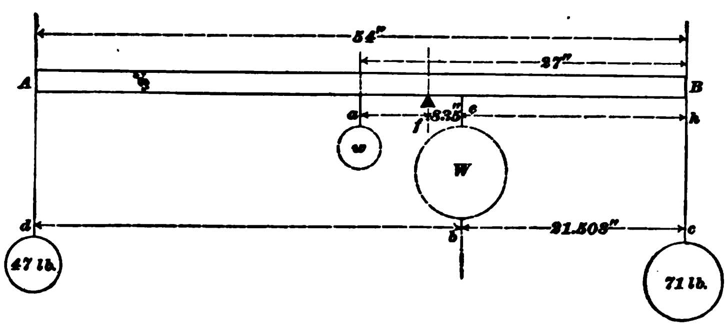

Fig. 22.

Consider both weights as concentrated at b, that is, imagine both weights removed and replaced by the dotted weight W, equal to $71 + 47 = 118$ lb. Consider the weight of the bar as concentrated at a, that is, as if replaced by a weight $w = 21.141$ lb. Then, the distance of the balancing point f, from e, or fe, $= \frac{21.141 \times 5.492}{21.141 + 118} = .834''$, since $ae = 27 - 21.508 = 5.492''$. Finally, $fe + eh = fh = .834 + 21.508 = 22.342'' =$ the short arm. Ans. $54 - 22.342 = 31.658'' =$ long arm. Ans.

(381) See Fig. 23.

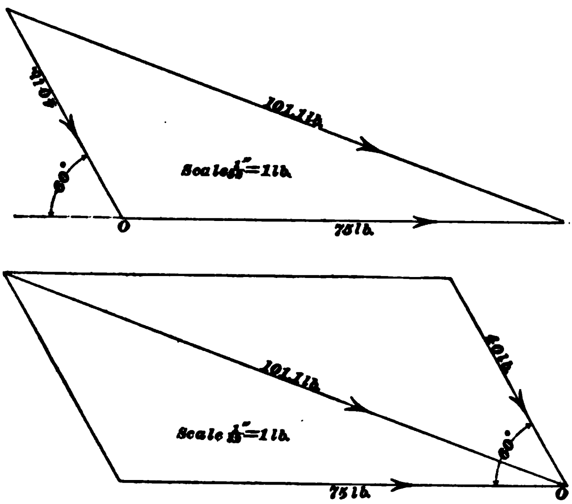

FIG. 23.

(382) See Fig. 24.

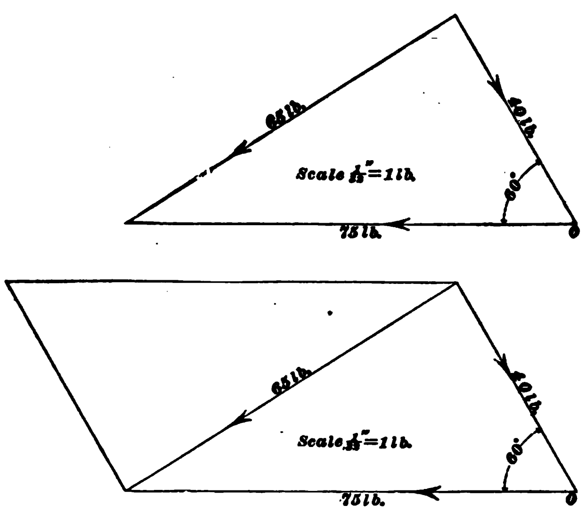

FIG. 24.

(383) $46 - 27 = 19$ lb., acting in the direction of the force of 46 lb. Ans.

(384) (*a*) $18 \times 60 \times 60 = 64{,}800$ miles per hour. Ans.
(*b*) $64{,}800 \times 24 = 1{,}555{,}200$ miles. Ans.

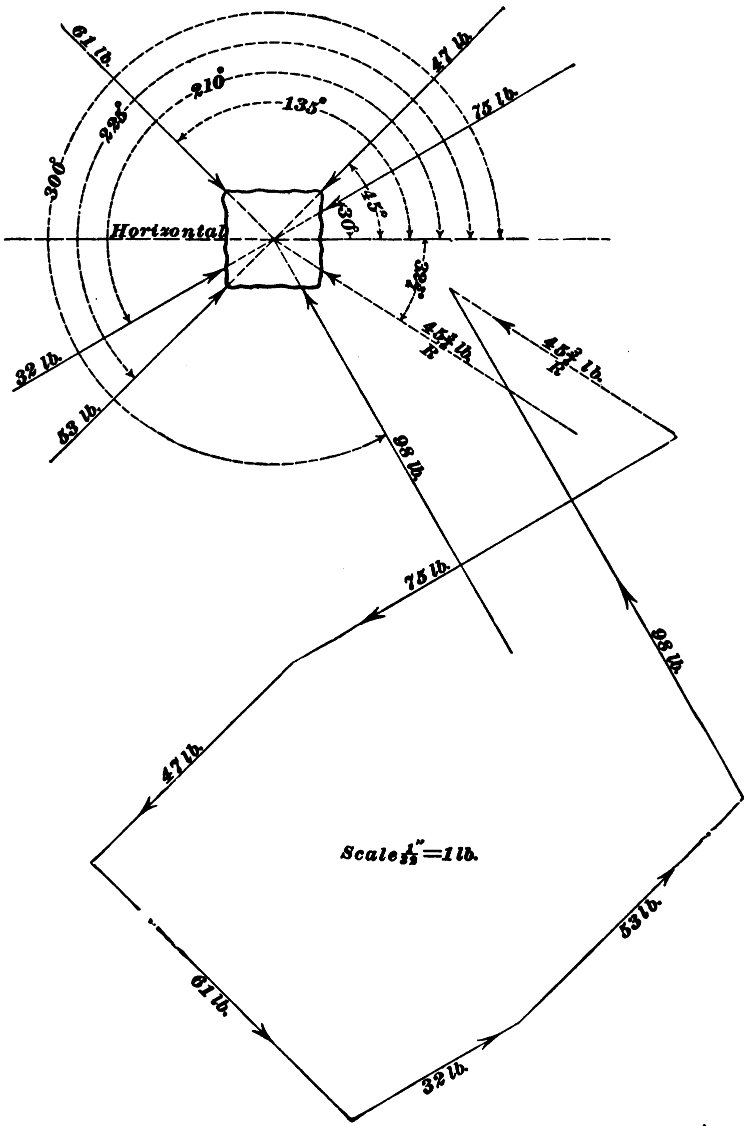

FIG. 25.

(385) (*a*) 15 miles per hour $= \frac{15 \times 5,280}{60 \times 60} = 22$ ft. per sec. As the other body is moving 11 ft. per sec., the distance between the two bodies in one second will be $22 + 11 = 33$ ft., and in 8 minutes the distance between them will be $33 \times 60 \times 8 = 15,840$ ft., which, divided by the number of feet in one mile, gives $\frac{15,840}{5,280} = 3$ miles. Ans.

(*b*) As the distance between the two bodies increases 33 ft. per sec., then, 825 divided by 33 must be the time required for the bodies to be 825 ft. apart, or $\frac{825}{33} = 25$ sec. Ans.

(386) See Fig. 25.

(387) (*a*) Although not so stated, the velocity is evidently considered with reference to a point on the shore. $10 - 4 = 6$ miles an hour. Ans.

(*b*) $10 + 4 = 14$ miles an hour. Ans.

(*c*) $10 - 4 + 3 = 9$, and $10 + 4 + 3 = 17$ miles an hour. Ans.

(388) See Fig. 26.

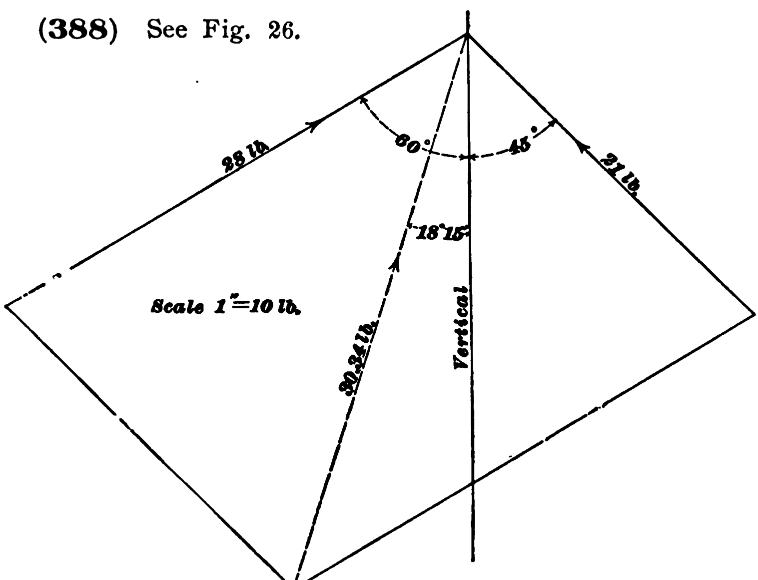

FIG. 26.

(389) See Fig. 27. By relations I and II, Art. **609,** $bc = 87 \sin 23° = 87 \times .39073 = 33.994$ lb., $ac = 87 \cos 23° = 87 \times .92050 = 80.084$ lb.

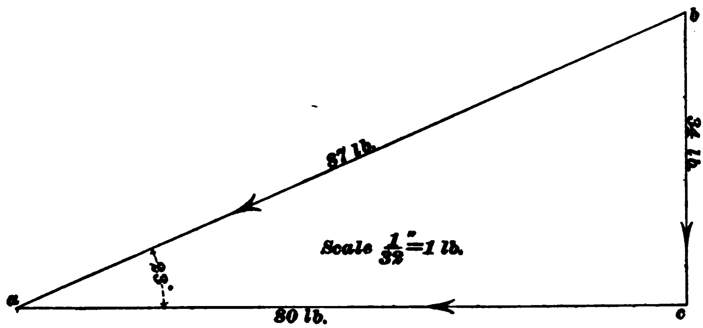

FIG. 27.

(390) See Fig. 28. (b) By relations I and II, Art. **609,** $bc = 325 \sin 15° = 325 \times .25882 = 84.12$ lb. Ans.

(a) $ac = 325 \cos 15° = 325 \times .96593 = 313.93$ lb. Ans.

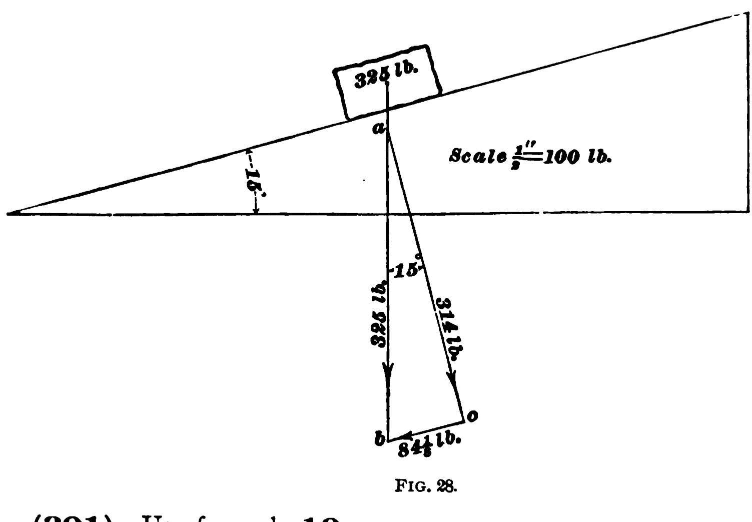

FIG. 28.

(391) Use formula **10.**

$$m = \frac{W}{g} = \frac{125}{32.16} = 3.8868. \quad \text{Ans.}$$

(392) Using formula **10,** $m = \frac{W}{g}$, $W = mg = 53.7 \times 32.16 = 1,727$ lb. Ans.

(393) (*a*) Yes. (*b*) 25. (*c*) 25. Ans.

(394) (*a*) Using formula **12,** $d^2 : R^2 :: W : w$, $d = \sqrt{\frac{R^2 W}{w}} = \sqrt{\frac{4,000^2 \times 141}{100}} = 4,749.736$ miles. $4,749.736 - 4,000 = 749.736$ miles. Ans.

(*b*) Using formula **11,** $R : d :: W : w$, $d = \frac{Rw}{W} = \frac{4,000 \times 100}{141} = 2,836.88$ miles. $4,000 - 2,836.88 = 1,163.12$ miles. Ans.

(395) (*a*) Use formula **18,**

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5,280}{32.16}} = 18.12 \text{ sec. Ans.}$$

(*b*) Use formula **13,** $v = gt = 32.16 \times 18.12 = 582.74$ ft. per sec., or, by formula **16,** $v = \sqrt{2gh} = \sqrt{2 \times 32.16 \times 5,280} = 582.76$ ft. per sec. Ans.

The slight difference in the two velocities is caused by not calculating the time to a sufficient number of decimal places, the actual value for t being 18.12065 sec.

(396) Use formula **25.** Kinetic energy $= Wh = \frac{Wv^2}{2g}$.

$$Wh = 160 \times 5,280 = 844,800 \text{ ft.-lb.}$$

$$\frac{Wv^2}{2g} = \frac{160 \times 582.76^2}{2 \times 32.16} = 844,799 \text{ ft.-lb. Ans.}$$

(397) (*a*) Using formulas **15** and **14,** $h = \frac{v^2}{2g} = \frac{2,360^2}{2 \times 32.16} = 86,592$ ft. $= 16.4$ miles. Ans. (*b*) $t = \frac{v}{g} =$ time required to go up or fall back. Hence, total time $= \frac{2v}{g}$ sec. $= \frac{2 \times 2,360}{60 \times 32.16} = 2.4461$ min. $= 2$ min. 26.77 sec. Ans.

(398) 1 hour = 60 min., 1 day = 24 hours; hence, 1 day = 60 × 24 = 1,440 min. Using formula **7**, $V = \frac{S}{t}$; whence, $V = \frac{8,000 \times 3.1416}{1,440} = 17.453 +$ miles per min. Ans.

(399) (*a*) Use formula **25**.

Kinetic energy $= \frac{W v^2}{2g} = \frac{400 \times 1,875 \times 1,875}{2 \times 32.16} = 21,863,339.55$ ft.-lb. Ans.

(*b*) $\frac{21,863,339.55}{2,000} = 10,931.67$ ft.-tons. Ans.

(*c*) See Art. **961**.

Striking force $\times \frac{6}{12} = 21,863,339.55$ ft.-lb.,

or striking force $= \frac{21,863,339.55}{\frac{6}{12}} = 43,726,679$ lb. Ans.

(400) Using formula **18**, $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 200}{32.16}} =$ 3.52673 sec., when $g = 32.16$.

$t = \sqrt{\frac{2 \times 200}{20}} = 4.47214$ sec., when $g = 20$.

$4.47214 - 3.52673 = 0.94541$ sec. Ans.

(401) See Art. **910**.

(402) See Art. **963**.

(403) (*a*) See Art. **962**.

$D = \frac{m}{V} = \frac{W}{gv}$. $v = \frac{800}{1,728}$. Hence, $D = \frac{W}{gv} = \frac{500}{32.16 \times \frac{800}{1728}} =$ 33.582. Ans. (*b*) In Art. **962**, the density of water was found to be 1.941. (*c*) In Art. **963**, it is stated that the specific gravity of a body is the ratio of its density to the density of water. Hence, $\frac{33.582}{1.941} = 17.3 =$ specific gravity. If the weight of water be taken as 62.5 lb. per cu. ft., the specific gravity will be found to be 17.28. Ans.

(404) Assuming that it started from a state of rest, formula **13** gives $v = g\,t = 32.16 \times 5 = 160.8$ ft. per sec.

(405) Use formulas **17** and **13.** $h = \frac{1}{2} g t^2 = \frac{32.16}{2} \times 3^2 = 144.72$ ft., distance fallen at the end of third second.

$v = g t = 32.16 \times 3 = 96.48$ ft. per sec., velocity at end of third second.

$96.48 \times 6 = 578.88$ ft., distance fallen during the remaining 6 seconds.

$144.72 + 578.88 = 723.6$ ft. = total distance. Ans.

(406) See Art. **961.**

Striking force $\times \frac{\frac{1}{2}}{12} = 8 \times 8 = 64$. Therefore, striking force $= \frac{64}{\frac{\frac{1}{2}}{12}} = 1{,}536$ tons. Ans.

(407) See Arts. **901** and **902.**

(408) Use formula **19.**

Centrifugal force = tension of string $= .00034\ W R N^2 = .00034 \times (.5236 \times 4^3 \times .261) \times \frac{15}{12} \times 60^2 = 13.38+$ lb. Ans.

(409) $(80^2 - 70^2) \times .7854 \times 26 \times .261 \div 2 =$ weight of $\frac{1}{2}$ of rim.

$$R = \frac{80 - 10}{2 \times 12} = \frac{35}{12} \text{ ft.}$$

According to Art. **904,** $F = .00034\ W R N^2 \div 3.1416 = .00034 \times \frac{(80^2 - 70^2) \times .7854 \times 26 \times .261}{2} \times \frac{35}{12} \times 175^2 \div 3.1416 = 38{,}641$ lb. Ans.

(410) (*a*) Use formulas **11** and **12.** $R : d :: W : w$, or $W = \frac{w R}{d} = \frac{1 \times 4{,}000}{100} = 40$ lb. Ans.

(*b*) $d^2 : R^2 :: W : w$, or $w = \frac{4{,}000^2 \times 40}{4{,}100^2} = 38.072$ lb. Ans.

(411) See Art. **955.**

$$\frac{10{,}746 \times 354}{10 \times 33{,}000} = 11.5275 \text{ H. P. Ans.}$$

(412) Use formula **12.**

$d^2 : R^2 :: W : w$, or $d = \sqrt{\dfrac{4{,}000^2 \times 2}{\frac{3}{16}}} = 13{,}064$ mi., nearly.

$13{,}064 - 4{,}000 = 9{,}064$ miles. Ans.

(413) Use formula **18.** $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2 \times 50}{32.16}} =$ 1.7634 sec., nearly.

$1.7634 \times 140 = 246.876$ ft. Ans.

(414) $\dfrac{10}{30} = \dfrac{1}{3}$ sec. Use formula **17.**

$h = \dfrac{1}{2} g t^2 = \dfrac{1}{2} \times 32.16 \times \left(\dfrac{1}{3}\right)^2 = 1.78\dfrac{2}{3}$ ft. $= 1$ ft. 9.44 in. Ans.

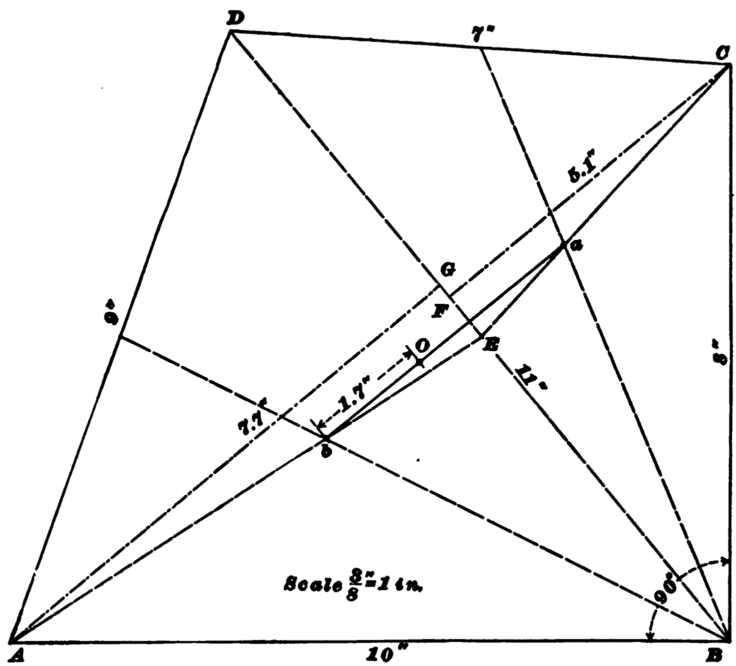

FIG. 29.

(415) See Arts. **906, 907.**

(416) See Arts. **908, 909.**

(417) No. It can only be counteracted by another equal couple which tends to revolve the body in an opposite direction.

(418) See Art. **914.**

(419) Draw the quadrilateral as shown in Fig. 29. Divide it into two triangles by the diagonal $B\ D$. The center of gravity of the triangle $B\ C\ D$ is found to be at a, and the center of gravity of the triangle $A\ B\ D$ is found to be at b (Art. **914**). Join a and b by the line $a\ b$, which, on being measured, is found to have a length of 4.27 inches. From C and A drop the perpendiculars $C\ F$ and $A\ G$ on the diagonal $B\ D$. Then, area of the triangle $A\ B\ D = \frac{1}{2}(A\ G \times B\ D)$, and area of the triangle $B\ C\ D = \frac{1}{2}(C\ F \times B\ D)$. Measuring these distances, $B\ D = 11''$, $C\ F = 5.1''$, and $A\ G = 7.7''$.

$$\text{Area } A\ B\ D = \frac{1}{2} \times 7.7 \times 11 = 42.35 \text{ sq. in.}$$

$$\text{Area } B\ C\ D = \frac{1}{2} \times 5.1 \times 11 = 28.05 \text{ sq. in.}$$

According to formula **20,** the distance of O, the center of gravity, from b is $\frac{28.05 \times 4.27}{28.05 + 42.35} = 1.7$. Therefore, the center of gravity is on the line $a\ b$ at a distance of $1.7''$ from b.

(420) See Fig. 30. The center of gravity lies at the geometrical center of the pentagon, which may be found as follows: From any vertex draw a line to the middle point of the opposite side. Repeat the operation for any other vertex, and the intersection of the two lines will be the desired center of gravity.

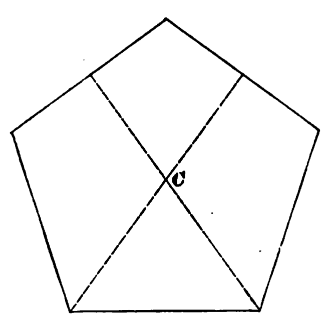

FIG. 30.

(421) See Fig. 31. Since any number of quadrilaterals can be drawn with the sides given, any number of answers can be obtained.

Draw a quadrilateral, the lengths of whose sides are equal to the distances between the weights, and locate a weight on each corner. Apply formula **20** to find the distance $C_1 W_1$; thus, $C_1 W_1 = \frac{9 \times 18}{9 + 21} = 5.4''$. Measure the distance $C_1 W_2$;

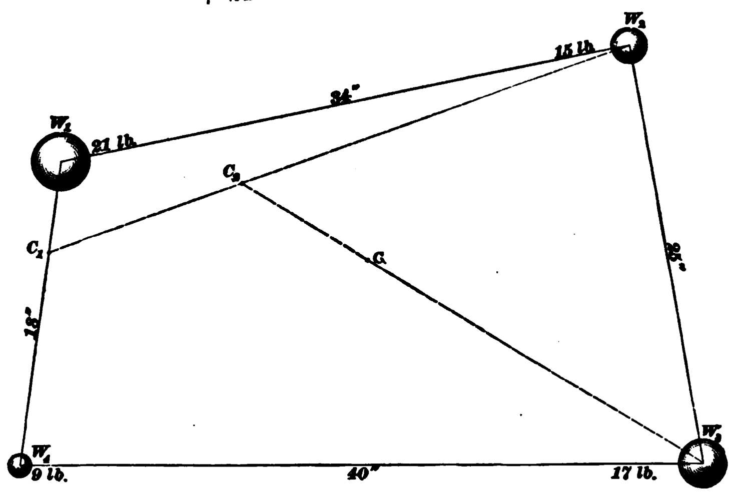

FIG. 31.

suppose it equals say 36″. Apply the formula again. $C_1 C_2 = \frac{15 \times 36}{15 + (9 + 21)} = 12''$. Measure $C_2 W_3$; it equals say 31.7″.

Apply the formula again. $C_2 C = \frac{17 \times 31.7}{17 + 15 + 9 + 21} = 8.7''$. C is center of gravity of the combination.

(422) Let $A\ B\ C\ D\ E$, Fig. 32, be the outline, the right-angled triangle cut-off being $E\ S\ D$. Divide the figure into two parts by the line $m\ n$, which is so drawn that it cuts off an isosceles right-angled triangle $m\ B\ n$, equal in area to $E\ S\ D$, from the opposite corner of the square.

The center of gravity of $AmnCDE$ is then at C_1, its geometrical center. $Bm = 4$ in.; angle $Bmr = 45°$; there-

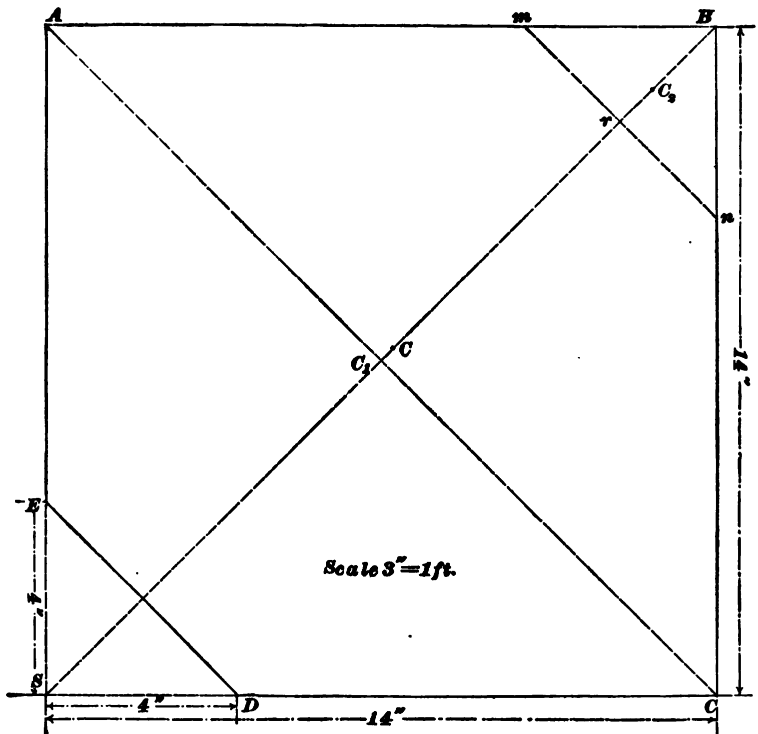

FIG. 32.

fore, $Br = Bm \times \sin Bmr = 4 \times .707 = 2.828$ in. C_2, the center of gravity of Bmn, lies on Br, and $BC_2 = \frac{2}{3}Br = \frac{2}{3} \times 2.828 = 1.885$ in. $BC_1 = AB \times \sin BAC_1 = 14 \times \sin 45° = 14 \times .707 = 9.898$ in. $C_1C_2 = BC_1 - BC_2 = 9.898 - 1.885 = 8.013$ in.

Area $ABCDE = 14^2 - \frac{4 \times 4}{2} = 188$ sq. in. Area $mBn = \frac{4 \times 4}{2} = 8$ sq. in. Area $AmnCDE = 188 - 8 = 180$ sq. in.

The center of gravity of the combined area lies at C, at

a distance from C_1, according to formula **20** (Art. **911**), equal to $\frac{8 \times C_1 C_2}{180 + 8} = \frac{8 \times 8.013}{188} = .341$ in. $C_1 C = .341$ in. $BC = BC_1 - C_1 C = 9.898 - .341 = 9.557$ inches. Ans.

(423) (*b*) In one revolution the power will have moved through a distance of $2 \times 15 \times 3.1416 = 94.248''$, and the weight will have been lifted $\frac{1''}{4}$. The velocity ratio is then

$94.248 \div \frac{1}{4} = 376.992$.

$376.992 \times 25 = 9{,}424.8$ lb. Ans.

(*a*) $9{,}424.8 - 5{,}000 = 4{,}424.8$ lb. Ans.

(*c*) $4{,}424.8 \div 9{,}424.8 = 46.95\%$. Ans.

(424) See Arts. **920** and **922**.

(425) Construct the prism *A B E D*, Fig. 33. From *E*, draw the line *E F*. Find the center of gravity of the

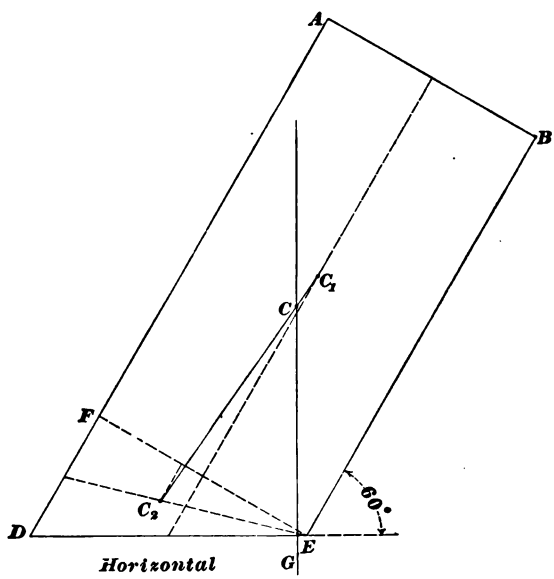

FIG. 33.

rectangle, which is at C_1, and that of the triangle, which is

at C_2. Connect these centers of gravity by the straight line $C_1 C_2$ and find the common center of gravity of the body by the rule to be at C. Having found this center, draw the line of direction $C G$. If this line falls within the base, the body will stand, and if it falls without, it will fall.

(426) (*a*) 5 ft. 6 in. = 66″. $66 \div 6 = 11$ = velocity ratio. Ans.

(*b*) $5 \times 11 = 55$ lb. Ans.

(427) $55 \times .65 = 35.75$ lb. Ans.

(428) Apply formula **20.** 5 ft. = 60″. $\frac{35 \times 60}{180 + 35} =$ 9.7674 in., nearly, = distance from the large weight. Ans.

(429) (*a*) $1{,}000 \div 50 = 20$, velocity ratio. Ans. See Art. **945.** (*b*) 10 fixed and 10 movable. Ans. (*c*) $50 \div 95 = 52.63\%$. Ans.

(430) $P \times$ circumference $= W \times \frac{1}{8}$, or $60 \times 40 \times 3.1416 = W \times \frac{1}{8}$, or $W = 60 \times 40 \times 3.1416 \times 8 = 60{,}318.72$ lb. Since the efficiency of combination is 40%, the tension on the stud would be $.40 \times 60{,}318.72 = 24{,}127.488$ lb. Ans.

(431) (*a*) $\sqrt{20^2 + 5^2} = 20.616$ ft. = length of inclined plane.

$P \times$ length of plane $= W \times$ height, or $P \times 20.616 = 1{,}580 \times 5$.

$P = \frac{1{,}580 \times 5}{20.616} = 383.2$ lb. Ans. (*b*) In the second case, $P \times$ length of base $= W \times$ height, or $P \times 20 = 1{,}580 \times 5$; hence, $P = \frac{1{,}580 \times 5}{20} = 395$ lb. Ans.

(432) $W \times 2 = 42 \times 6$, or $W = \frac{42 \times 6}{2} = 126$ lb.

$126 + 42 = 168$ lb. $168 \times 1 = W' \times 12$, or $W' = \frac{168}{12} = 14$ lb. Ans.

(433) See Fig. 34. $P \times 14 \times 21 \times 19 = 2\frac{1}{2} \times 3\frac{1}{4} \times 2\frac{7}{8} \times 725$, or

$$P = \frac{2\frac{1}{2} \times 3\frac{1}{4} \times 2\frac{7}{8} \times 725}{14 \times 21 \times 19} = 3.032 \text{ lb. Ans.}$$

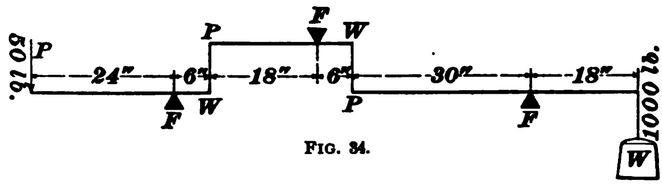

FIG. 34.

(434) See Fig. 35. (*a*) $35 \times 15 \times 12 \times 20 = 5 \times 3\frac{1}{2} \times 3 \times W$, or

$$W = \frac{35 \times 15 \times 12 \times 20}{5 \times 3\frac{1}{2} \times 3} = 2{,}400 \text{ lb. Ans.}$$

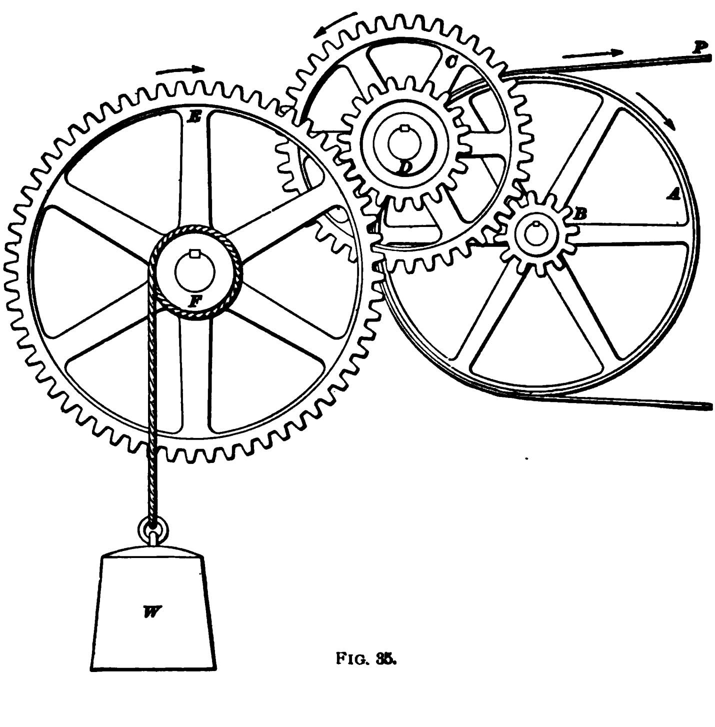

FIG. 35.

(*b*) $2,400 \div 35 = 68\frac{4}{7}$ = velocity ratio. Ans.

(*c*) $1,932 \div 2,400 = .805 = 80.5\%$. Ans.

(435) In Fig. 36, let the 12-lb. weight be placed at A; the 18-lb. weight at B, and the 15-lb. weight at D.

Use formula **20.**

$\frac{12 \times 15}{18 + 12} = 6'' =$ distance $C_1 B$ = distance of center of

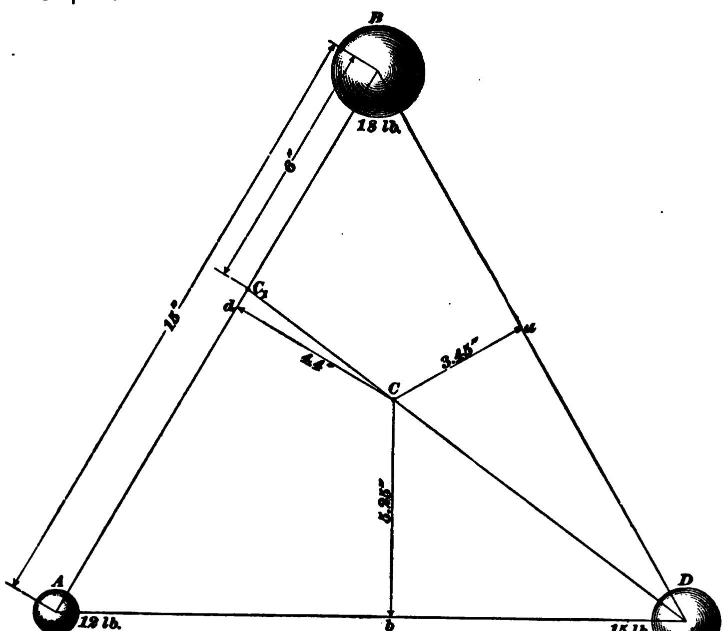

FIG. 36.

gravity of the 12 and 18-lb. weights from B. Drawing $C_1 D$, $C_1 C = \frac{15 \times C_1 D}{(12 + 18) + 15} = \frac{1}{3} C_1 D$. Measuring the distances of C from BD, DA, and AB, it is found that $Ca = 3.45''$, $Cb = 5.25''$, and $Cd = 4.4''$. Ans.

(436) (*a*) Potential energy equals the work which the body would do in falling to the ground $= 500 \times 75 =$ 37,500 ft.-lb. Ans.

(*b*) Using formula **18**, $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 75}{32.16}} = 2.1597$ sec. $= .035995$ min., the time of falling.

$$\frac{37,500}{33,000 \times .035995} = 31.57 \text{ H. P.} \quad \text{Ans.}$$

(437) $127 \div 62.5 = 2.032 =$ specific gravity. Ans.

(438) $\frac{62.5}{1,728} \times 9.823 = .35529$ lb. Ans.

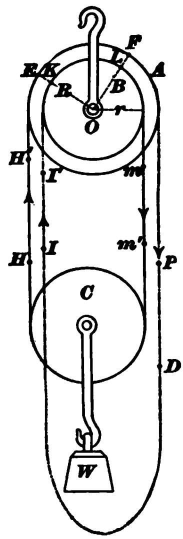

FIG. 37.

(439) Use formula **21**. $W = \left(\frac{2PR}{R-r}\right)$, or $\frac{2 \times 60 \times 6.5}{6.5 - 5.75} \times .48 = 499.2$ lb. Ans. See Fig. 37.

(440) See Art. **961**.

$$F \times \left(\frac{3}{8} \div 12\right) = \frac{Wv^2}{2g} = \frac{1.5 \times 25^2}{2 \times 32.16}, \text{ or}$$

$$F = \frac{\frac{1.5 \times 25^2}{2 \times 32.16}}{\frac{3}{8} \div 12} = 466.42 \text{ lb.} \quad \text{Ans.}$$

(441) (*a*) $2,000 \div 4 = 500 =$ wt. of cu. ft. $500 \div 62.5 = 8 =$ specific gravity. Ans.

(*b*) $\frac{500}{1,728} = .28935$ lb. Ans.

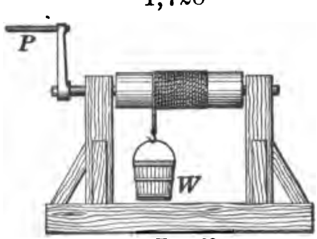

FIG. 38.

(442) See Fig. 38. $14.5 \times 2 = 29$. $30 \times 29 = W \times 5$, or $W = \frac{30 \times 29}{5} = 174$ lb. Ans.

(443) $75 \times .21 = 15.75$ lb. Ans.

(444) (*a*) $900 \times 150 = 135,000$ ft.-lb. Ans.

$$\frac{135,000}{15} = 9,000 \text{ ft.-lb. per min.} \quad \text{Ans.}$$

(*b*) $\frac{9,000}{33,000} = \frac{3}{11}$ H.P. Ans.

(445) $900 \times .18 \times 2 = 324$ lb. = force required to overcome the friction. $900 + 324 = 1{,}224$ lb. = total force.

$$\frac{1{,}224 \times 150}{15 \times 33{,}000} = .37091 \text{ H. P.} \quad \text{Ans.}$$

(446) $18 \div 88 = .2045$. Ans.

(447) See Art. **962.** $D = \dfrac{W}{g\,V} = \dfrac{1{,}200}{32.16 \times 3} = 12.438$. Ans.

(448) See Fig. 39. $125 - 47.5 = 77.5$ lb. = downward pressure.

$77.5 \div 4 = 19.375$ lb. = pressure on each support. Ans.

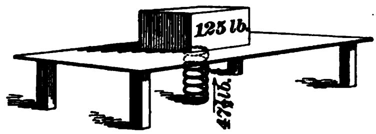

FIG. 39.

(449) See Fig. 40.

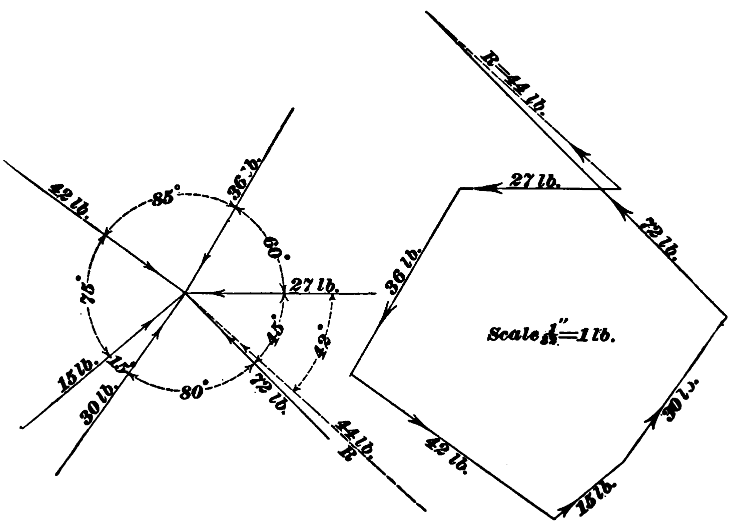

FIG. 40.

(450) See Fig. 41. $4.5 \div 2 = 2.25$.

$$\frac{12}{2.25} \times 6 \times 30 = 960 \text{ lb.} \quad \text{Ans.}$$

(451) (*a*) $960 \div 30 = 32$. Ans.

(*b*) $790 \div 960 = .8229 = 82.29\%$. Ans.

(452) (*a*) See Fig. 42. $475 + (475 \times .24) = 589$ lb. Ans

(*b*) $475 \div 589 = .8064 = 80.64\%$. Ans.

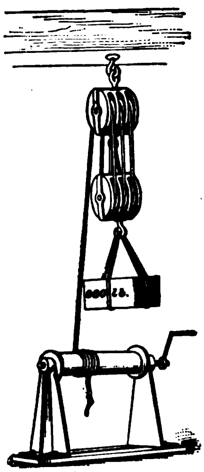

FIG. 41.

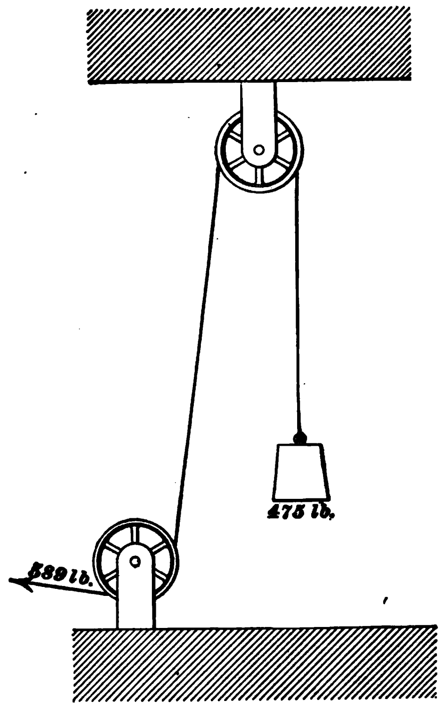

FIG. 42.

(453) (*a*) By formula **23,** $U = FS = 6 \times 25 = 150$ foot-pounds. Ans.

$$(b)\quad 2\frac{1}{2} \text{ sec.} = \frac{2\frac{1}{2}}{60} = \frac{1}{24} \text{ min.}$$

Using formula **24,** Power $= \frac{FS}{T} = \frac{150}{\frac{1}{24}} = 3{,}600$ ft.-lb. per min. Ans.

PRINCIPLES OF ELECTRICITY AND MAGNETISM.

(QUESTIONS 1079–1187.)

(1079) See Art. **2240.**

(1080) One joule $= .7373$ foot-pound (Art. **2333**); therefore, electrical energy =

$$\frac{\text{foot-pounds}}{.7373} = \frac{123,750}{.7373} = 167,842.1266 \text{ joules.}$$

Electrical power (Art. **2349**) =

$$\frac{\text{energy}}{\text{time in seconds}} = \frac{167,842.1266}{60 \times 43.5} = 64.3073 \text{ watts. Ans.}$$

(1081) Let C_1 = current in A and C_2 = current in B. According to Art. **2322**, the conductivity of $A = \frac{1}{16.2}$ and of $B = \frac{1}{14.1}$. Therefore (Art. **2323**), $C_1 : C_2 :: \frac{1}{16.2} : \frac{1}{14.1}$, or $\frac{C_1}{14.1} = \frac{C_2}{16.2}$; hence, $C_1 = \frac{14.1\, C_2}{16.2}$.

Now, $C_1 + C_2 = 6.37$, or $C_1 = 6.37 - C_2$. Substituting, $6.37 - C_2 = \frac{14.1\, C_2}{16.2}$, or $103.194 - 16.2\, C_2 = 14.1\, C_2$; hence, $30.3\, C_2 = 103.194$, and $C_2 = 3.4057$ amperes.

$$C_1 = 6.37 - 3.4057 = 2.9643 \text{ amperes.}$$

Therefore, the current in branch $A = 2.9643$ amperes. Ans.
Therefore, the current in branch $B = 3.4057$ amperes. Ans.

(1082) (*a*) Power, or rate of expending energy = $\frac{\text{energy}}{\text{time}} = \frac{99,370,000}{2.5} = 39,748,000$ ergs per second. From

Art. **2264**, 1 watt = 10,000,000 ergs per second; therefore,

$$\frac{\text{ergs per second}}{10{,}000{,}000} = \frac{39{,}748{,}000}{10{,}000{,}000} = 3.9748 \text{ watts.} \quad \text{Ans.}$$

(*b*) $\text{Horsepower} = \dfrac{\text{watts}}{746} = \dfrac{3.9748}{746} = .005328 \text{ H. P.}$ Ans.

(1083) Using formula **433**,

$$\text{Ampere-turns} = \frac{N}{3.192} \times \left(\frac{l_1}{A_1 \times \mu_1} + \frac{l_2}{A_2 \times \mu_2} + \frac{2\,l_3}{A_3 \times \mu_3}\right).$$

$l_1 = 2.5 \times 2 \times 3.1416 = 15.708''$. $A_1 = 2 \times 1 = 2$ sq. in.

$B_1 = \dfrac{N}{A_1} = \dfrac{160{,}000}{2} = 80{,}000$ lines of force per sq. in.; from Table 82, $\mu = 769.2$ in wrought iron at that density. *The reluctance of the magnetic circuit in the wrought iron* =

$$\frac{l_1}{A_1 \times \mu_1} = \frac{15.708}{2 \times 769.2} = .01021.$$

$l_2 = 4 \times 1 = 4''$. $A_2 = 2 \times 2 = 4$ sq. in.

$B_2 = \dfrac{160{,}000}{4} = 40{,}000$ lines of force per sq. in.; from Table 79, $\mu = 152.9$ in cast iron at that density. *The reluctance of the magnetic circuit in the cast iron* =

$$\frac{l_2}{A_2 \times \mu_2} = \frac{4}{4 \times 152.9} = \frac{1}{152.9} = .00654.$$

$l_3 = .125''$. $A_3 = 2 \times 2 = 4$ sq. in. $\mu_3 = 1$. *The reluctance of the magnetic circuit in two air-gaps* =

$$\frac{2\,l_3}{A_3 \times \mu_3} = \frac{2 \times .125}{4 \times 1} = .0625.$$

The total reluctance of the magnetic circuit = .01021 + .00654 + .0625 = .07925.

$$\text{Ampere-turns} = \frac{160{,}000}{3.192} \times .07925 = \frac{12{,}680}{3.192} = 3{,}972.431.$$

Ans.

(1084) (*a*) Use formula **438**, and let P = the tractive force in pounds exerted by one contact surface; $A = 2 \times 2 = 4$ sq. in., the area of one contact surface; and $\mathbf{B} = \dfrac{N}{A} = \dfrac{240{,}000}{4} = 60{,}000$ lines of force per sq. in., the magnetic density of contact surface. Substituting,

$$P = \frac{60,000 \times 60,000 \times 4}{72,134,000} = 199.6284 \text{ lb.},$$

tractive force of one polar surface; total tractive force $=$ 199.6284 $\times$ 2 $=$ 399.2568 lb. Ans.

(*b*) Use formula **442** given in Art. **2429**, $p = \frac{\mathbf{B}^2}{72,134,000}$, and let p = pounds per sq. in. of contact surface and $\mathbf{B}$ = 60,000 lines of force per sq. in., the density at contact surface. Substituting, $p = \frac{60,000 \times 60,000}{72,134,000} = 49.9071$ lb. per sq. in. Ans.

(1085) From the Greek word *Elektron*, meaning *amber*.

(1086) (Formula **409.**)

$$C = \frac{E}{R} = \frac{20}{30 + 80} = .1818 \text{ ampere.} \quad \text{Ans.}$$

(1087) One joule $=$ .7373 foot-pound; therefore, electrical energy $= \frac{\text{foot-pounds}}{.7373} = \frac{742,500}{.7373} = 1,007,052.76$ joules. Power is the *rate* of expending energy, and is found by dividing energy by the time. (Art. **2349.**)

$$\text{Electrical power} = \frac{\text{joules}}{\text{seconds}} = \frac{1,007,052.76}{(4 \times 3,600) + (21 \times 60)} =$$

$$\frac{1,007,052.76}{15,660} = 64.3073 \text{ watts.} \quad \text{Ans.}$$

(1088) The sectional area of A is $.02^2 \times .7854 = .00031416$ sq. in. The sectional area of B is $.02 \times .02 = .0004$ sq. in. The conductivity of a conductor is proportional to its sectional area. Therefore, the relative conductivities of the two branches A and B are .00031416 and .0004, respectively. The current divides among the branches of a divided circuit in proportion to their conductivities. Therefore, if C_1 represents the current in branch A, and C_2 the current in branch B, then $C_1 : C_2 :: .00031416 : .0004$; or, $C_1 : C_2 ::$ $.7854 : 1$; hence, $C_2 = \frac{C_1}{.7854}$, or $C = .7854\, C_2$. Now, $C_1 +$

$C_2 = 2.6$, or $C_1 = 2.6 - C_2$. Equating the two values of C_1, $2.6 - C_2 = .7854\,C_2$, or $C_2 = \frac{2.6}{1.7854} = 1.4562$ amperes in B. Ans.

$C_1 = 2.6 - 1.4562 = 1.1438$ amperes in A. Ans.

(1089) (Art. **2272.**) 1 absolute unit of current = 10 amperes; consequently, 6.74 absolute units = $6.74 \times 10 =$ 67.4 amperes. Ans.

(1090) See Art. **2201.**

(1091) (Formula **409.**)

$$C = \frac{E}{R} = \frac{112}{49.2} = 2.2764 \text{ amperes.} \quad \text{Ans.}$$

(1092) (Formula **419.**)

$$W = E\,C = 112.5 \times 12.2 = 1{,}372.5 \text{ watts.} \quad \text{Ans.}$$

(1093) (Art. **2323.**) Let C_1 = current in A and C_2= current in B; the conductivity of $A = \frac{1}{\text{resistance}} = \frac{1}{49.2}$; the conductivity of $B = \frac{1}{\text{resistance}} = \frac{1}{67}$. The current divides among the branches of a derived circuit in proportion to the conductivities of the branches; therefore, $C_1 : C_2 ::$ $\frac{1}{49.2} : \frac{1}{67}$, or $\frac{C_1}{C_2} = \frac{67}{49.2}$; hence, $C_1 = \frac{67\,C_2}{49.2}$. But $C_1 + C_2 = .76$, or $C_1 = .76 - C_2$. Substituting, $.76 - C_2 = \frac{67\,C_2}{49.2}$, or $116.2\,C_2 =$ 37.392; therefore, $C_2 = .3218$ ampere, the current in branch B. Ans.

$C_1 = .76 - C_2 = .76 - .3218 = .4382$ ampere, the current in A. Ans.

(1094) See Art. **2303.** 1 volt = 100,000,000 absolute units of potential; therefore, 1,764,300,000 absolute units of potential $= \frac{1{,}764{,}300{,}000}{100{,}000{,}000} = 17.643$ volts. Ans.

(1095) From Table 83 (Art. **2414**) the loss is .01518 watt per cubic inch for 1 cycle, when the density is 90,300

lines of force per sq. in. 1 cycle = 2 reversals; therefore, 133 reversals = 66.5 cycles.

Watts expended = number of cycles × cubic inches of iron × .01518 = 66.5 × 65 × .01518 = 65.61555 watts. Ans.

(1096) See Arts. **2205, 2275,** and **2383.**

(1097) (Art. **2310,** formula **411.**) $E = C R = (36.2 + 21.7) \times .127 = 57.9 \times .127 = 7.3533$ volts. Ans.

(1098) By formula **420** the power in watts is $W = C^2 R = 19.8^2 \times 11.8 = 392.04 \times 11.8 = 4,626.072$ watts. Ans.

(1099) (Formula **412.**) Joint resistance $= R' = \frac{r_1 r_2}{r_2 + r_1} = \frac{7.19 \times .92}{.92 + 7.19} = \frac{6.6148}{8.11} = .8156$ ohm. Ans.

(1100) By formula **413** the joint resistance in parallel is

$$R''' = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} =$$

$$\frac{37.3 \times 49.9 \times 57.1}{(49.9 \times 57.1) + (37.3 \times 57.1) + (37.3 \times 49.9)} =$$

$$\frac{106,278.517}{2,849.29 + 2,129.83 + 1,861.27} = \frac{106,278.517}{6,840.39} = 15.537 \text{ ohms.}$$

Ans.

(1101) (Art. **2282.**) One ohm = 1,000,000,000 absolute units of resistance; therefore, 875,000,000 absolute units $= \frac{875,000,000}{1,000,000,000} = .875$ ohm. Ans.

(1102) (Art. **2419.**) 165,750 − 115,300 = 50,450 lines of force which are not useful. According to formula **436,** the per cent. leakage $= p = \frac{50,450}{165,750} \times 100 = 30.437\%$. Ans.

(1103) *Indestructibility* and *elasticity.*

(1104) (Formula **411.**)

$E = C R = 2.73 \times 49.3 = 134.589$ volts. Ans.

(1105) (Formula **421.**)

$W = \frac{E^2}{R} = \frac{78.8^2}{2.3} = \frac{6,209.44}{2.3} = 2,699.7565$ watts. Ans.

(1106) (Formula **412.**) Joint resistance in parallel is

$$R' = \frac{r_1 r_2}{r_2 + r_1} = \frac{42.1 \times 98.3}{98.3 + 42.1} = \frac{4{,}138.43}{140.4} = 29.476 \text{ ohms. Ans.}$$

(1107) See Art. **2358.**

(1108) By transposition of terms in formula **437,** the number of useful lines of force is

$$l_u = \frac{l\,(100 - p)}{100} = \frac{376{,}420 \times (100 - 33)}{100} = 252{,}201.4$$

useful lines of force. Ans.

(1109) Static charges and currents.

(1110) (Formula **410.**) $R = \frac{E}{C} = \frac{22.4}{.43} = 52.093$ ohms, total resistance of the circuit. The external resistance is the difference between the total resistance and the internal, or $52.093 - 13.4 = 38.693$ ohms. Ans.

(1111) (Formula **422.**) 1 horsepower = 746 watts; therefore, $2.33 \times 746 = 1{,}738.18$ watts. Ans.

(1112) Let r_1 = resistance of branch A and r_2 = resistance of branch B. Their joint resistance in parallel, by formula **412,** is

$$R' = \frac{r_1 r_2}{r_2 + r_1}. \quad \text{Then, } 19.9 = \frac{r_1 r_2}{r_2 + r_1}, \text{ or } r_1 = \frac{19.9\, r_2}{r_2 - 19.9}.$$

Let $C_1 = 2.2$ amperes, the current in branch A; and $C_2 = 5.9$ amperes, the current in B. By Art. **2323,**

$$C_1 : C_2 :: \frac{1}{r_1} : \frac{1}{r_2}.$$

Substituting,

$$2.2 : 5.9 :: \frac{1}{\dfrac{19.9\, r_2}{r_2 - 19.9}} : \frac{1}{r_2}, \text{ or } \frac{5.9\,(r_2 - 19.9)}{19.9\, r_2} = \frac{2.2}{r_2},$$

and, $5.9\, r_2 - 117.41 = 43.78$, or $5.9\, r_2 = 161.19$, whence $r_2 = 27.3203$ ohms, the resistance of branch B. Ans.

$C_1 : C_2 :: \frac{1}{r_1} : \frac{1}{r_2}$, or $2.2 : 5.9 :: \frac{1}{r_1} : \frac{1}{27.3203}$, whence $2.2\,r_1 = 5.9 \times 27.3203$, or $r_1 = \frac{161.18977}{2.2} = 73.2681$ ohms, the resistance of branch A. Ans.

(1113) See Art. **2363.**

(1114) (Formula **437.**) The total number of lines of force is

$$l = \frac{100\,l_u}{100 - p} = \frac{100 \times 1{,}764{,}300}{100 - 61} = \frac{176{,}430{,}000}{39} = 4{,}523{,}846$$

lines of force developed by magnetizing coils. Ans.

(1115) See Art. **2204.**

(1116) (*a*) (Formula **410.**) $R = \frac{E}{C} = \frac{28.2}{5.2} = 5.423$ ohms. Ans.

(*b*) Let r = internal resistance; then, $7\,r$ = the external resistance (the external being 7 times the internal). $r + 7\,r = 5.423$, or $8\,r = 5.423$, and $r = .677875$ ohm, internal resistance. Ans. $7\,r = 7 \times .677875 = 4.745125$ ohms, external resistance. Ans.

(1117) (Formula **422.**) 746 watts = 1 horsepower; therefore, 63,426 watts $= \frac{63{,}426}{746} = 85.0214$ horsepower. Ans.

(1118) Let C_1 = current in branch A; C_2 = current in branch B; and r = resistance of branch B. Then, $4.5\,r$ = resistance of branch A, since the resistance of branch A is 4.5 times the resistance of branch B. (Art. **2323.**) $C_1 : C_2 :: \frac{1}{4.5\,r} : \frac{1}{r}$, or $C_1 = \frac{C_2}{4.5}$. (The two r's cancel.) Also, $C_1 = 23.4 - C_2$, since the sum of the currents in the two branches equals 23.4 amperes. Equating, $23.4 - C_2 = \frac{C_2}{4.5}$, or $105.3 - 4.5\,C_2 = C_2$, whence $105.3 = C_2 + 4.5\,C_2$, or $5.5\,C_2 = 105.3$, and $C_2 = 19.1454$ amperes, current in B. Ans.

$C_1 = 23.4 - C_2 = 23.4 - 19.1454 = 4.2546$ amperes, current in A. Ans.

(1119) See Art. **2366.**

(1120) See Art. **2208.**

(1121) A strength of current of 1 ampere decomposes .0014388 grain of water in 1 second (Art. **2277**); to find the total amount of water, in grains, decomposed, multiply .0014388 by the time in seconds and then by the strength of current in amperes. $.0014388 \times 60 \times 60 = 5.17968$. $5.17968 \times 10 = 51.7968$ grains of water. Ans.

(1122) (Art. **2355.**) 1 horsepower = 746 watts; therefore, 83.42 horsepower $= 83.42 \times 746 = 62{,}231.32$ watts. Ans.

(1123) Let $r =$ the resistance of branch B; then, $2.7\,r =$ the resistance of branch A, since the resistance of branch A is 2.7 times the resistance of branch B. (Art. **2325.**) Joint resistance $= \frac{r \times 2.7\,r}{r + 2.7\,r}$; therefore, $47.9 = \frac{r \times 2.7\,r}{r + 2.7\,r}$. Hence, $2.7\,r = 3.7 \times 47.9$, or $2.7\,r = 177.23$ ohms, the resistance of branch A. Ans.

$r = \frac{177.23}{2.7} = 65.6407$ ohms, the resistance of branch B. Ans.

(1124) See Art. **2366.**

(1125) See Art. **2438.**

(1126) (*a*) Negative. (*b*) Negative. (*c*) Positive.

(1127) (Formula **402.**) The current $C = \frac{w}{.0014388 \times t}$; where $w =$ weight of water in grains, and $t =$ time of flow in seconds. Substituting, $C = \frac{25}{.0014388 \times 4 \times 60 \times 60} = 1.2066$ amperes. Ans.

(1128) (Formula **423.**)

$$\text{Horsepower} = \frac{W}{746} = \frac{3{,}362}{746} = 4.5067. \quad \text{Ans.}$$

(1129) (Formula **412.**) Joint resistance in parallel $=$

$$R' = \frac{r_1 r_2}{r_2 + r_1} = \frac{2.4 \times 987.3}{987.3 + 2.4} = \frac{2,369.52}{989.7} = 2.394 \text{ ohms. Ans.}$$

(1130) (Formula **427.**)

$$\mathbf{B} = \frac{N}{A} = \frac{103,750}{2.25 \times 1.75} = \frac{103,750}{3.9375} = 26,349.2 \text{ lines of force}$$

per square inch. Ans.

(1131) See Art. **2441.**

(1132) To detect static charges of electricity and determine their sign, whether *positive* or *negative.*

(1133) The total resistance of the circuit $= 17.2 + 8.2 + 11.3 = 36.7$ ohms. (Formula **411.**) $E = C R = .75 \times 36.7 = 27.525$ volts, total electromotive force generated in the battery. Ans.

(Formula **411.**) $E = C R = .75 \times 11.3 = 8.475$ volts, difference of potential between a and b. Ans.

$E = C R = .75 \times 8.2 = 6.15$ volts, difference of potential between b and c. Ans.

Art. **2318.** $E' = E - C r_1 = 27.525 - (.75 \times 17.2) = 14.625$ volts, difference of potential between c and a.

(1134) (Formula **424.**) Mechanical power, or H. P. $=$

$$\frac{E C}{746} = \frac{225 \times 86.3}{746} = 26.0288 \text{ horsepower. Ans.}$$

(1135) The resistances may be found by Ohm's law, as follows:

The difference of potential is 11.6 volts $= E$; the current in A is 6.7 amperes $= C$; therefore, $R = \frac{E}{C} = \frac{11.6}{6.7} = 1.7313$ ohms. Likewise for branch B, the resistance $R = \frac{11.6}{4.9} = 2.3673$ ohms. Ans.

This example may also be worked out in the following manner:

The sum of the currents in the two branches is $6.7 + 4.9 = 11.6$ amperes. By formula **410**, $R = \frac{E}{C} = \frac{11.6}{11.6} = 1$ ohm, the joint resistance of the two branches in parallel. Let $r_1 =$ the separate resistance of branch A and $r_2 =$ the separate resistance of branch B. Then, by formula **412**, their joint resistance $= R'' = \frac{r_1 r_2}{r_2 + r_1}$. Therefore, since 1 ohm is also their joint resistance, $1 = \frac{r_1 r_2}{r_2 + r_1}$, or $r_2 = \frac{r_1}{r_1 - 1}$.

Let $C_1 =$ the current in branch A, and C_2 the current in branch B.

Then, $C_1 : C_2 :: \frac{1}{r_1} : \frac{1}{r_2}$.

Substituting, $6.7 : 4.9 :: \frac{1}{r_1} : \frac{1}{\frac{r_1}{r_1 - 1}}$, or $\frac{4.9}{r_1} = \frac{6.7 r_1 - 6.7}{r_1}$.

Multiplying each side by r_1, $4.9 = 6.7\ r_1 - 6.7$, or $6.7\ r_1 = 4.9 + 6.7$; whence $r_1 = 1.7313$ ohms, the resistance of branch A. Ans.

$C_1 : C_2 :: \frac{1}{r_1} : \frac{1}{r_2}$. Substituting, $6.7 : 4.9 :: \frac{1}{1.7313} : \frac{1}{r_2}$, or $4.9\ r_2 = 11.59971$, and $r_2 = 2.3673$ ohms, resistance of branch B. Ans.

(1136) The sectional area of the magnet = diameter$^2 \times .7854 = .5^2 \times .7854 = .25 \times .7854 = .19635$ sq. in. The magnetic density is found by dividing the total number of lines of force passing through a magnetic circuit or a magnet by the sectional area; therefore, by formula **427**, $\mathbf{B} = \frac{3,927}{.19635} = 20,000$ lines of force per sq. in. Ans.

(1137) (Art. **2449.**) An electromotive force of 1 volt is generated by a conductor when it is cutting lines of force at the rate of 100,000,000 per second; therefore, by formula **447**, $E = \frac{N}{100,000,000 \times t} = \frac{3,660,000,000}{100,000,000 \times 60} = .61$ volt. Ans.

(1138) See Art. **2211.**

(1139) (Art. **2292.**) The resistance of a portion of the conductor is, by formula **406,**

$$r_2 = \frac{r_1 \times l_2}{l_1} = \frac{12.6 \times 11}{16} = 8.6625 \text{ ohms.}$$

From formula **411,** $E = C R = 2.3 \times 8.6625 = 19.92375$ volts, difference of potential between the end and the point. Ans.

(1140) (Art. **2356.**) Mechanical power =

$$\frac{C^2 R}{746} = \frac{11.4^2 \times 12.3}{746} = 2.1427 \text{ horsepower. Ans.}$$

(1141) First find the joint resistance between a and b, or, in other words, the joint resistance of the two branches in parallel, by formula **412** (Art. **2324**). The joint resistance is $R'' = \frac{r_1 r_2}{r_2 + r_1} = \frac{793 \times 979.3}{979.3 + 793} = \frac{776,584.9}{1,772.3} = 438.1791$ ohms.

From formula **411,** $E = C R = .23 \times 438.1791 = 100.781193$ or 100.7812 volts, difference of potential between a and b. Ans.

(1142) The sectional area of the bar $= .25 \times .375 = .09375$ sq. in. The total number of lines of force is found by multiplying the sectional area by the magnetic density; therefore, by formula **428,** $N = .09375 \times 34,500 = 3,234.375$ lines of force. Ans.

(1143) By formula **447,**

$$E = \frac{N}{100,000,000\, t} = \frac{48,900,000,000}{100,000,000 \times 60} = 8.15 \text{ volts. Ans.}$$

(1144) If 4 units of electricity repel the lever arm of a torsion balance through an angle of 20 degrees, then it is evident that the force that would be required to repel the lever arm of the torsion balance through an angle of 1 degree would be only $\frac{1}{20^2}$ as great, that is, $\frac{1}{20^2} \times 4 = \frac{4}{400}$ $= .01$ of a unit of electricity. The quantity of electricity

required to repel the lever arm through a distance of $45° = 45^2 \times .01 = 45 \times 45 \times .01 = 20.25$ units of electricity. Ans.

(1145) Reduce the time to seconds. $60 \times 60 \times 4.5 = 16,200$ seconds.

By transposition of formula **405**,

$$C = \frac{Q}{t} = \frac{368,422}{16,200} = 22.7421 \text{ amperes. Ans.}$$

(1146) (*a*) (Formula **409.**)

$$C = \frac{E}{R} = \frac{48.4}{.23} = 210.4348 \text{ amperes. Ans.}$$

(*b*) (Formula **421.**)

Electrical power $W = \frac{E^2}{R} = \frac{48.4^2}{.23} = 10,185.04$ watts. Ans.

(*c*) (Formula **423.**)

Mechanical power $= \frac{W}{746} = \frac{10,185.04}{746} = 13.6528$ horsepower. Ans.

(1147) This problem can be solved by the simple application of *Ohm's law*, and by considering each branch separately. (Art. **2310.**)

In branch A, the difference of potential between the two ends is 125 volts, and the resistance of the branch is 47.41 ohms; consequently by formula **409**, the current $= \frac{E}{R} = \frac{125}{47.41} = 2.6366$ amperes. Ans.

In branch B, the difference of potential between the two ends is also 125 volts, and the resistance is 69.8 ohms; consequently, $C = \frac{E}{R} = \frac{125}{69.8} = 1.7908$ amperes. Ans.

(1148) The sectional area of the bar $= 2^2 \times .7854 = 3.1416$ sq. in. Applying formula **428**, $N = A\,\mathbf{B} = 3.1416 \times 56,000 = 175,929.6$ lines of force. Ans.

(1149) (*a*) Since the force exerted between two statically charged bodies is equal to the product of their two respective charges (Art. **2214**), and a unit quantity of electricity

attracts another unit quantity of opposite sign with a force of 1 dyne at a distance of 1 centimeter, then the force of attraction exerted between the two gilt balls at a distance of 1 centimeter is $20 \times 5 = 100$ dynes. Ans.

(*b*) According to the law of attraction and repulsion of static charges (Art. **2213**), the force exerted between them varies inversely as the square of the distance; consequently, if $f =$ the force at 5 centimeters, then $100 : f :: 5^2 : 1$, or $5^2 f = 100$, or $f = 4$ dynes. Ans.

(1150) First reduce the time to seconds. $60 \times 60 \times 2.25 = 8{,}100$ seconds. From Art. **2281**, formula **405**, $Q = C\,t = 8.32 \times 8{,}100 = 67{,}392$ coulombs. Ans.

(1151) (Art. **2356**.) By formula **424**, the mechanical power =

$$\frac{E\,C}{746} = \frac{525 \times 12.5}{746} = \frac{6{,}562.5}{746} = 8.7969 \text{ horsepower. Ans.}$$

(1152) (Art. **2326**.) The joint resistance of three branches in parallel is

$$R''' = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} =$$

$$\frac{7.9 \times 13.3 \times 25.8}{(13.3 \times 25.8) + (7.9 \times 25.8) + (7.9 \times 13.3)} =$$

$$\frac{2{,}710.806}{343.14 + 203.82 + 105.07} = \frac{2{,}710.806}{652.03} = 4.1575 \text{ ohms. Ans.}$$

(1153) The end *b*; because in looking at that end the current is circulating around the solenoid in the opposite direction to the motion of the hands of a watch. (Art. **2390**.)

(1154) As soon as the two gilt balls come in contact, the $+4$ charge on ball A neutralizes a -4 charge on ball B, leaving a negative charge of -20 units, which immediately divides equally between the two balls. A charge of -10 units on each ball produces a repulsion of $10 \times 10 = 100$ dynes between the two balls at a distance of 1 centimeter; and at a distance of 2 centimeters the force will be

$\frac{100}{2^2} = 25$ dynes, since the force exerted between them varies inversely as the square of the distance. (Art. **2213.**)

(1155) Transposing formula **405,** $t = \frac{Q}{C} = \frac{86,700}{10} =$ 8,670 seconds, or 2 hours 24 minutes and 30 seconds. Ans.

(1156) (Art. **2356.**) Mechanical power = $\frac{C^2 R}{746} = \frac{46.8^2 \times 4.4}{746} = \frac{9,637.056}{746} = 12.9183$ horsepower. Ans.

(1157) Towards the East.

(1158) See Art. **2224.**

(1159) From Art. **2333,** 1 joule = .7373 foot-pound; therefore, 1 foot-pound = $\frac{1}{.7373} = 1.3563$ joules. Ans.

(1160) (Formula **426.**) Mechanical power = $\frac{E^2}{746\,R} = \frac{225.6^2}{746 \times 15.7} = \frac{50,895.36}{11,712.2} = 4.3455$ horsepower. Ans.

(1161) From formula **413,** the joint resistance of the three branches in parallel =

$$\frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} =$$

$$\frac{63.2 \times 76.5 \times 88.9}{(76.5 \times 88.9) + (63.2 \times 88.9) + (63.2 \times 76.5)} =$$

$$\frac{429,813.72}{6,800.85 + 5,618.48 + 4,834.8} = \frac{429,813.72}{17,254.13} = 24.9108 \text{ ohms.}$$

The difference of potential between where the three branches divide and where they unite is $C \times R = 5.7 \times 24.9108 = 141.99156$ volts.

To find the current in each branch, consider each separately and apply *Ohm's law* (formula **409**):

The current in branch A is $\frac{E}{r_1} = \frac{141.99156}{63.2} = 2.2467$ amperes. Ans.

The current in branch B is $\frac{E}{r_2} = \frac{141.99156}{76.5} = 1.8561$ amperes.
Ans.

The current in branch C is $\frac{E}{r_3} = \frac{141.99156}{88.9} = 1.5972$ amperes.
Ans.

(1162) The current is flowing *from* the north *towards* the south.

(1163) Positive. Since a negative charge is developed on sulphur when rubbed with silk (Art. **2209**), this negative charge will induce a *bound* positive charge on the cover towards the sulphur and a free negative charge upon the opposite side of the cover, which will neutralize with the earth and leave the positive charge alone upon the cover.

(1164) (*a*) First reduce the time to seconds. $1.25 \times 60 \times 60 = 4{,}500$ seconds. By formula **414,** electrical energy $= J = C^2 R t = 14.2 \times 14.2 \times 8 \times 4{,}500 = 7{,}259{,}040$ joules. Ans.

(*b*) By formula **417,** mechanical energy in foot-pounds F. P. $= .7373 J = .7373 \times 7{,}259{,}040 = 5{,}352{,}090.192$ foot-pounds. Ans.

(1165) Let r_1 = resistance of 1 mile of copper wire, and r_2 = resistance of 1 mile of iron wire. Since the resistances of two metals are inversely proportional to their respective conductivities, $r_1 : r_2 :: 1 : 6.08$, or $r_2 = 6.08\ r_1$. Substituting 14 for r_1, $r_2 = 6.08 \times 14 = 85.12$ ohms per mile, the resistance of the iron wire. As the resistance of the conductor is proportional to its length (Art. **2292**), then, by formula **406,** the resistance of 1,000 feet of the iron wire is

$$r_2 = \frac{85.12 \times 1{,}000}{5{,}280} = 16.1212 \text{ ohms. Ans.}$$

NOTE.—In the two cases, as the diameters of the wires are equal, their sectional areas are also equal; therefore, the sectional areas can be neglected in solving the problem. See Art. **2293.**

(1166) The sum of the currents in the three branches is $7.7 + 23.9 + 15 = 46.6$ amperes. The difference of potential between the point where the branches divide and the

point where they unite is by formula **411** $= CR = 46.6 \times 3.2 = 149.12$ volts. To find the resistance of each branch, consider each branch separately and apply *Ohm's law;* therefore (Art. **2310**),

The separate resistance of branch A is

$$\frac{E}{C_1} = \frac{149.12}{7.7} = 19.3662 \text{ ohms. Ans.}$$

The separate resistance of branch B is

$$\frac{E}{C_2} = \frac{149.12}{23.9} = 6.2393 \text{ ohms. Ans.}$$

The separate resistance of branch C is

$$\frac{E}{C_3} = \frac{149.12}{15} = 9.9413 \text{ ohms. Ans.}$$

(1167) From formula **430,** the intensity of magnetomotive force is $\mathbf{H} = \frac{3.192 \times a\text{-}t}{l} = \frac{3.192 \times 4{,}620}{10} = 1{,}474.704$. Ans.

A magnetic density of 1,474.704 lines of force is produced in a magnetic circuit when $\mathbf{H} = 1{,}474.704$; therefore, the total number of lines of force in the circuit is equal to the product of the magnetic density and the sectional area, or $1{,}474.704 \times 3.2 = 4{,}719.0528$ lines of force. Ans.

(1168) The area of the surface of the sphere = diameter$^2 \times 3.1416 = 4 \times 4 \times 3.1416 = 50.2656$ sq. in. From Art. **2225,** the electric density of a charged body is found by dividing the number of units of electricity by the area of the surface upon which it resides; therefore, the electric density of the sphere is $400 \div 50.2656 = 7.9577$ units of electricity per sq. in. Ans.

(1169) From Art. **2333,** 1 joule = .7373 foot-pound; therefore, 5,326,824 foot-pounds are equivalent to $\frac{5{,}326{,}824}{.7373} = 7{,}224{,}771.463$ joules. Electrical energy in joules = EQ; therefore, $Q = \frac{\text{joules}}{E} = \frac{7{,}224{,}771.463}{220} \doteq 32{,}839.8703$

coulombs. Transposing formula **405**,

$$C = \frac{Q}{t} = \frac{32{,}839.8703}{60 \times 60 \times 2.5} = 3.6489 \text{ amperes. Ans.}$$

(1170) From Art. **2217**, the conductivity is the reciprocal of the resistance; therefore, the conductivity of copper $= \frac{1}{1} = 1$, whence the conductivity of mercury $= \frac{1}{59}$ or .01695. Ans.

(1171) (Art. **2326.**) The joint resistance of the three branches in parallel, by formula **413** $= \dfrac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} = \dfrac{.2 \times 1.9 \times 2.1}{1.9 \times 2.1 + .2 \times 2.1 + .2 \times 1.9} = \dfrac{.798}{3.99 + .42 + .38} = \dfrac{.798}{4.79} =$ .1666 ohm. The difference of potential between a and b is, therefore (formula **411**), $E = CR = 67.4 \times .1666 = 11.22884$ volts. Ans.

(1172) From formula **430**, the intensity of magnetomotive force $= \mathbf{H} = \dfrac{3.192 \times a\text{-}t}{l}$; transposing,

$$\textit{ampere-turns} = \frac{\mathbf{H} \times l}{3.192}.$$

Substituting the values of **H** and l,

$$\textit{ampere-turns} = \frac{360 \times 12}{3.192} = 1{,}353.383. \quad \text{Ans.}$$

(1173) *Negative;* because the negative charge on the inside coating induces a *bound* positive charge upon the inner face of the outside coating and leaves a free negative charge upon the outer face of the outside coating.

(1174) First reduce the time to seconds; $15 \times 60 =$ 900 seconds. Then, transposing formula **417**, $J = \dfrac{\text{F. P.}}{.7373} = \dfrac{984{,}650}{.7373} = 1{,}335{,}480.808$ joules. Electrical energy in joules $=$ $E\,Q$; therefore, $Q = \dfrac{\text{joules}}{E} = \dfrac{1{,}335{,}480.808}{115} = 11{,}612.8766$

coulombs of electricity. By transposition of formula **405**, $C = \frac{Q}{t} = \frac{11,612.8766}{900} = 12.9032$ amperes; then, applying Ohm's law, $R = \frac{E}{C} = \frac{115}{12.9032} = 8.9125$ ohms. Ans.

(1175) From Art. **2286**, the resistance of a column of mercury 41.7323 in. high and .00155 sq. in. sectional area is 1 ohm. The resistance of any conductor is directly proportional to the length of the conductor, Art. **2293**; then, if the column were 1,000 feet high, its resistance r_1, is given by the proportion $41.7323 : 12,000 :: 1 : r_1$. Equating, $41.7323\ r_1 = 12,000$, or $r_1 = 287.54705$ ohms. If the sectional area of the column were $.02^2 \times .7854 = .00031416$ sq. in., instead of .00155 sq. in., its resistance r_2 would then be represented by the proportion $287.54705 : r_2 :: .00031416 : .00155$; since the resistance of any conductor is inversely proportional to its sectional area. (Art. **2295.**) Equating, $.00031416\ r_2 = .00155 \times 287.54705$, or $r_2 = 1,418.69724$ ohms. If the resistance of mercury is 62.73 times the resistance of silver, then the resistance of a silver wire 1,000 feet long and .02″ in diameter would be

$$\frac{1,418.69724}{62.73} = 22.6159 \text{ ohms. Ans.}$$

(1176) This problem is a simple application of *Ohm's law* as given in Art. **2310**; each branch can be considered as a separate conductor having a difference of potential of 225 volts between its two ends.

Let C_1 = the current in branch A,
C_2 = the current in branch B;
C_3 = the current in branch C.

The separate resistance of branch A is $\frac{E}{C_1} = \frac{225}{17} = 13.2353$ ohms. Ans.

The separate resistance of branch B is $\frac{E}{C_2} = \frac{225}{39} = 5.7692$ ohms. Ans.

The separate resistance of branch C is $\frac{E}{C_3} = \frac{225}{23} = 9.7826$ ohms. Ans.

(1177) From Art **2400,** $\mu = \frac{\mathbf{B}}{\mathbf{H}}$; where $\mu =$ the permeability of the substance, $\mathbf{B} =$ the magnetic density, and $\mathbf{H} =$ the intensity of magnetomotive force. Substituting for $\mathbf{B}$ and $\mathbf{H}$, $\mu = \frac{54,300}{600} = 90.5$. Ans.

(1178) See Art. **2237.**

(1179) See Art. **2349.**

(1180) From Art. **2286,** the resistance of a column of mercury 41.7323 in. high and .00155 sq. in. sectional area is 1 ohm; if the column were 72.3 in. high, the resistance (r_1) is given by the proportion $41.7323 : 72.3 :: 1 : r_1$, since the resistance of any conductor is directly proportional to its length. (Art. **2293.**) Equating, $41.7323\, r_1 = 72.3$, or $r_1 = 1.7324$ ohms. If the sectional area be decreased to $.04^2 \times .7854 = .00125664$ sq. in., the resistance (r_2) will be given by the proportion $1.7324 : r_2 :: .00125664 : .00155$; since the resistance of any conductor is inversely proportional to its sectional area. (Art. **2295.**) Equating, $.00125664\, r_2 = 1.7324 \times .00155$, or $r_2 = 2.1368$ ohms. Ans.

(1181) Each branch can be considered as a separate conductor, with a difference of potential of 108 volts between its two ends; the current in each branch is found by the application of *Ohm's law*, as in Art. **2310.**

Let $r_1 =$ the resistance of branch A;
$r_2 =$ the resistance of branch B;
$r_3 =$ the resistance of branch C.

The current in branch A is $\frac{E}{r_1} = \frac{108}{52.2} = 2.069$ amperes. Ans.

The current in branch B is $\frac{E}{r_2} = \frac{108}{73} = 1.4795$ amperes. Ans.

The current in branch C is $\frac{E}{r_3} = \frac{108}{64} = 1.6875$ amperes. Ans.

(1182) From Art. **2400,** $\mathbf{H} = \frac{\mathbf{B}}{\mu}$; where μ = the permeability, **H** = the intensity of the magnetomotive force, and **B** = the magnetic density. Substituting 850 for μ and 59,500 for **B**, $\mathbf{H} = \frac{59,500}{850} = 70$. Ans.

(1183) (1) Contact of two dissimilar metals. (2) Chemical action. (3) Magnetic induction.

(1184) From Art. **2357,** 746 watts = 1 horsepower; therefore, 1 watt = $\frac{1}{746}$ horsepower = .00134 horsepower. Ans.

(1185) Since the resistance (Art. **2293**) of any conductor is directly proportional to its length, the resistance of 2,000 feet of the round wire would be $1 \times \frac{2,000}{1,000} = 2$ ohms. The sectional area of the round wire is $.1^2 \times .7854 = .007854$ sq. in., and the sectional area of the square wire is $0.1 \times 0.1 = .01$ sq. in. Since the resistance of any conductor is inversely proportional to its sectional area (Art. **2295**), then the resistance (r) of 2,000 feet of square wire is given by the proportion $2 : r :: .01 : .007854$; whence, equating, $.01\,r = 2 \times .007854$, or $r = 1.5708$ ohms. Ans.

(1186) Since 1 joule = 10,000,000 ergs (Art. **2330**), then 600,000,000 ergs are equivalent to $\frac{600,000,000}{10,000,000} = 60$ joules. From Art. **2333,** 1 joule = .7373 foot-pound; then, 60 joules are equivalent to $60 \times .7373 = 44.238$ foot-pounds. Ans.

(1187) From Art. **2400,** $\mathbf{B} = \mu\,\mathbf{H}$; where **B** = the magnetic density, μ = the permeability of the magnetic substance, and **H** = the intensity of magnetomotive force. Substituting 180 for **H** and 450 for μ, $\mathbf{B} = 450 \times 180 = 81,000$. Ans.

ELECTRICAL MEASUREMENTS.

(QUESTIONS 1188–1229.)

(1188) The multiplying power of a shunt of 500 ohms resistance used with a galvanometer of 690 ohms resistance is (Art. **2488**)

$$n + 1 = \frac{R_g}{R_s} + 1.$$

Substituting the values of R_s and R_g given in this example,

$$n + 1 = \tfrac{690}{500} + 1 = 2.38.$$

The current flowing through the galvanometer is, from formula **454,**

$$C_g = \frac{C}{n + 1}.$$

Substituting the values of C and $n + 1$ in the above formula,

$$C_g = \frac{.06}{2.38} = .02521 \text{ ampere},$$

the current in the galvanometer when shunted.

As a current of .06 ampere gives a deflection of 68° with this galvanometer, its constant may be calculated from formula **450,** $C = K \tan m°$.

Substituting the above values,

$$.06 = K \times 2.4751,$$

or $$K = \frac{.06}{2.4751} = .024242.$$

A current of .02521 ampere will then give

$$.02521 = .024242 \times \tan m^\circ;$$

whence, $$\tan m^\circ = \frac{.02521}{.024242} = 1.0399.$$

From the table of tangents, $m^\circ = 46^\circ\ 7'+$. Ans.

(1189) (*a*) Use formula **448**, $f = \frac{2\pi A t}{r}$.

In this example, $A = .1$, $t = 1$, $r = 12$ cm. Substituting these values,

$$f = \frac{2\pi \times .1 \times 1}{12} = \frac{.62832}{12} = .05236 \text{ dyne.} \quad \text{Ans.}$$

(*b*) In Washington, the value of the horizontal component of the earth's magnetism H is .200. (Table 84.) Then, by formula **449**, $f = H \tan m^\circ$. Substituting the above values, $f = .05236$ and $H = .200$,

$$.05236 = .200 \times \tan m^\circ,$$

or $$\tan m^\circ = \frac{.05236}{.200} = .2618.$$

From the table of tangents, $m^\circ = 14^\circ\ 40'+$. Ans.

(1190) A controlling magnet serves to vary the action of the earth's field on the magnet of the galvanometer, by increasing or decreasing the strength of the earth's field at that point. (Art. **2466.**)

(1191) The specific resistance of mica is 84 tregohms (Table 85); that is, the resistance between two opposite faces of a cube of mica, each edge being 1 cm. long, is 84 tregohms. 1 cm. = .3937 inch; therefore, the resistance of a piece of mica $\frac{1}{8}$ inch long and 1 cm. square would be, calculated from formula **406**,

$$r_2 = \frac{r_1 \times l_2}{l_1} = \frac{84 \times .125}{.3937} = 26.67 \text{ tregohms.}$$

1 sq. cm. = $.3937 \times .3937 = .155$ sq. in. 26.67 tregohms is, then, the resistance of a piece of mica .155 sq. in. in sectional area, .125 inch long. To find the resistance of a

piece $10 \times 10 = 100$ square inches area, formula **407**, $r_2 = \frac{r_1 a_1}{a_2}$, may be used,

$$r_2 = \frac{26.67 \times .155}{100} = .04134 \text{ tregohm.}$$

From Art. **2515** the prefix *trega* implies a unit 1,000,000,000,000 times as large as the ordinary unit, so in this case, to reduce .04134 tregohm to ohms, multiply .04134 by 1,000,000,000,000, which gives 41,340,000,000 ohms.

Ans.

(1192) The diagram should be about like Fig. 97. The principle of the bridge is described in Art. **2509.**

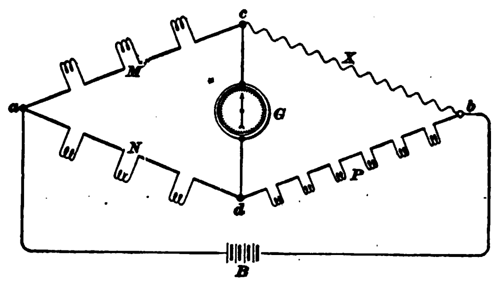

FIG. 97.

(1193) (*a*) In example 1189, a current of 1 ampere through 1 turn (section III) of the coil gave a value of $f = .05236$ dyne; a current of 4.2 amperes through 3 turns (section II) of the coil would then give a value of $f = 4.2 \times 3 \times .05236 = .65974$ dyne.

As this force gives a deflection of 40°, the value of H may be found from formula **449**, $f = H \tan m°$.

$$\text{Tan } m° = \tan 40° = .8391;$$

hence, $$.65974 = H \times .8391,$$

or $$H = \frac{.65974}{.8391} = .78625.$$

The value of H due to the earth's field at Philadelphia is .194. (Table 84.) Consequently, this field is increased $.78625 - .194 = .5923$ line of force per sq. cm. Ans.

(*b*) If the polarity of the controlling magnet be reversed, the earth's field will be lessened by .5923 line of force per sq. cm.; or $.194 - .5923 = -.3983$ line of force per sq. cm.; that is, the value of H acting on the galvanometer needle will be entirely due to the controlling magnet, and will be in a direction opposite to the lines of force of the earth's field. The deflection will therefore be in the opposite direction, and its amount may be found from formula **449**, $f = H \tan m°$, by substituting the values of f and H.

$f = .65974$, as before, the current being the same. $H = .3983$, as found above. (The minus sign merely indicates *direction* of the field and may be omitted.)

Hence, $.65974 = .3983 \times \tan m°$, or

$$\tan m° = \frac{.65974}{.3983} = 1.6564.$$

Then, from the table of tangents, $m° = 58° \, 53'$. Ans.

Being in the opposite direction, this means that the *difference* between the two deflections is $58° \, 53' + 40° = 98° \, 53'$.

(1194) (*a*) See Art. **2504.**
(*b*) See Art. **2504.**
(*c*) See Art. **2542.**

(1195) (*a*) See Art. **2475.**
(*b*) See Art. **2538.**

(1196) See Art. **2458.**

(1197) The principle of this method of measuring insulation resistance is given in Art. **2530.** The result may be found from formula **465**, $I = \frac{R\,x\,d°}{d_1°}$. As in this case $d°$ and $d_1°$ are identical, $I = R\,x$. The number of cells $x = 100$, and R is 11,800 ohms. Then $I = 11{,}800 \times 100 = 1{,}180{,}000$ ohms, the total insulation resistance of the wire. This sample of wire is 20 feet long, but 1 foot of each end projects from the liquid, so that the actual length of the piece tested is 18 feet, or 6 yards. The total

insulation resistance multiplied by the length in yards gives the insulation resistance per yard of length. (Art. **2523.**)

$$6 \times 1,180,000 = 7,080,000 \text{ ohms per yard.}$$

Dividing this result by 1,000,000 gives the megohms per yard, or

$$\frac{7,080,000}{1,000,000} = 7.08 \text{ megohms per yard. Ans.}$$

(1198) See Art. **2476.**

(1199) (Art. **2513.**) Applying formula **459** for working up the readings of the slide-wire bridge,

$$X = \frac{M}{N} \times P =$$

$$\frac{n\,b}{a\,n} \times P = \frac{73.8}{26.2} \times 10 = 28.17 \text{ ohms. Ans.}$$

(1200) This may be found from formula **448,**

$$f = \frac{2\,\pi\,A\,t}{r}.$$

In this example, $A = \frac{2.6}{10} = .26$; $t = 2$; $r = 11$; $2\,\pi = 6.2832$.

Substituting these values in the above formula,

$$f = \frac{6.2832 \times .26 \times 2}{11} = \frac{3.2673}{11} = .29703 \text{ dyne. Ans.}$$

(1201) To find the galvanometer constant from the deflection, the strength of the current flowing must first be determined. This may be found from the increase in weight of the copper negative plate (Art. **2496**), which is 2.26 grams. As is there stated, a current of 1 ampere will deposit .0003286 gram of copper in 1 second. Then, if 2.26 grams of copper have been deposited in 1 hr. 40 min., or $(60 + 40) \times 60 = 6,000$ seconds, $\frac{2.26}{6,000} = .00037667$ gram was deposited each second. As 1 ampere would have deposited .0003286 gram in the same time, the actual current must have been $\frac{.00037667}{.0003286} = 1.1463$ amperes. Knowing the current

and the deflection it produces, the galvanometer constant may be calculated from formula **450,** $C = K \tan m^\circ$.

Substituting the values of C and $\tan m^\circ$ in this equation,

$$1.1463 = K \times .9004, \text{ or } K = \frac{1.1463}{.9004} = 1.2731. \quad \text{Ans.}$$

(1202) By the electromagnetic effect. (Art. **2459.**)

(1203) (*a*) The specific resistance of a saturated solution of copper sulphate is 29.3 ohms. (Table 86.) The length and sectional area of the body of copper sulphate in this example being known, the actual resistance of the body may be calculated from formulas **406** and **407,** as follows:

The resistance of a body of the liquid 1 cm. long and 1 sq. cm. sectional area is 29.3 ohms, from the table.

The length of the liquid in the example is 4 inches, or $4 \times 2.54 = 10.16$ cm. Then, the resistance of a body of this liquid 1 cm. in sectional area, 10.16 cm. long, would be

$$r_2 = \frac{r_1 \times l_2}{l_1}.$$

Substituting the values of r_1, l_1, and l_2,

$$r_2 = \frac{29.3 \times 10.16}{1} = 297.7 \text{ ohms.}$$

The sectional area of the body of liquid in this example is, however, 6 in. $\times$ 8 in. = 48 sq. in. = $48 \times 6.45 = 309.6$ sq. cm., and the resistance of this body, 10.16 cm. long and 309.6 sq. cm. area, may be found from the formula

$$r_2 = \frac{r_1 a_1}{a_2}.$$

Substituting the values of r_1, a_1, and a_2,

$$r_2 = \frac{297.7 \times 1}{309.6} = .9616 \text{ ohm.} \quad \text{Ans.}$$

(*b*) From the table, the specific resistance of a solution of sal ammoniac of maximum conductivity is 2.50 ohms. If the copper sulphate, specific resistance 29.3 ohms, be replaced with the sal ammoniac, specific resistance 2.50 ohms, the resistance of the body will be reduced in the

same proportion; that is, the resistance with sal ammoniac will be $\frac{2.50}{29.3} = .085324$ times that with copper sulphate.

Then, $.9616 \times .085324 = .082047$ ohm, resistance of sal ammoniac solution. Ans.

(1204) The multiplying power of a shunt is $n + 1$, where $n = \frac{R_g}{R_s}$, R_g being the resistance of the galvanometer and R_s the resistance of the shunt. (Art. **2488.**)

In this case,

(*a*) $R_g = 120$ ohms, $R_s = 20$;
then, $n = \frac{120}{20} = 6$, and $n + 1 = 6 + 1 = 7$. Ans.

(*b*) $R_g = 180$ ohms, $R_s = 20$;
then, $n = \frac{180}{20} = 9$, and $n + 1 = 9 + 1 = 10$. Ans.

(*c*) $R_g = 300$, $R_s = 20$;
then, $n = \frac{300}{20} = 15$, and $n + 1 = 15 + 1 = 16$. Ans.

(1205) See Art. **2509.**

(*a*) In this case, the various circuits of the bridge would be as represented in Fig. 98.

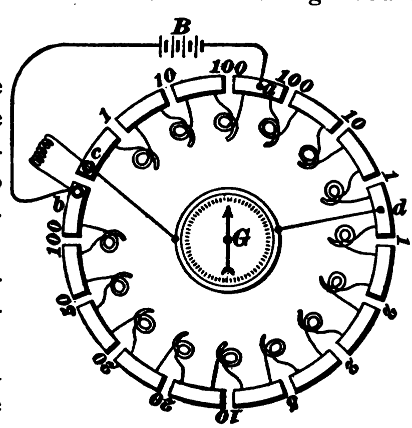

FIG. 98.

The balance arms are *a c* and *a d*; adjustable arm, *d b*. *G* is the galvanometer, connected to *c* and *d*, and *B* is the battery, connected to *a* and *b*. The unknown resistance is connected between *b* and *c*.

(*b*) The resistances in each balance arm are equal, being $100 + 10 + 1 = 111$ ohms each. The resistance in the adjustable arm is $1 + 2 + 2 + 5 + 10 + 20 + 20 + 50 + 100 = 210$ ohms. The unknown resistance is then found from formula **459,**

$$X = \frac{M}{N} \times P.$$

In this case, $M = 111$, $N = 111$, and $P = 210$.

Then, $X = \frac{111}{111} \times 210 = 210$ ohms. Ans.

(1206) The resistance coils of the bridge, having no temperature coefficient, would always be of the same resistance; but the resistance of the copper wire coil would vary with the temperature. (Art. **2516.**) In this example, the resistance of the copper wire coil, as measured by the bridge, is 210 ohms at 91° F. At 45° F., which is a temperature of 46° F. lower, its resistance would be found to be less. According to formula **463,**

$$r_2 = \frac{r_1}{1 + bF^\circ} = \frac{210}{1 + .002156 \times 46^\circ} = \frac{210}{1.099176} = 191.05.$$

The change in resistance is the difference between 210, its first value, and 191.05, its decreased value, or

$$210 - 191.05 = 18.95 \text{ ohms. Ans.}$$

(1207) (*a*) See Art. **2520.**
(*b*) See Art. **2521.**

(1208) (*a*) A diagram similar to that shown in Fig. 991 is required.

(*b*) The calibrating coil is used to set up a field, the intensity of which can be calculated. Since this field is set up in air, wood, or other non-magnetic medium, the intensity of the field, **H**, in the coil may be calculated by formula **430,** because the number of turns on the solenoid and also its axial length are known. The kick of the galvanometer corresponding to various known values of **H** can thus be calculated, and by comparing these kicks with those obtained when the secondary coil S on the iron ring is in use, the value of **B** may be determined as explained in Art. **2486.**

(*c*) The objection to the step-by-step method is that if any mistake is made in one of the first readings, it is carried through the whole series of readings. If careful work is

done, however, this objection should not prove serious. See Art. **2486.**

(*d*) In the step-by-step method, the current in the primary is suddenly increased or decreased. In the reversal method, the current in the primary is reversed by means of the switch *H*, Fig. 991. See Arts. **2484** and **2485.**

(1209) (*a*) The heating effect of the current is made to cause the expansion of a wire. This expansion is then measured, giving a measure of the current.

(*b*) The Cardew voltmeter is one. (Art. **2541.**)

(1210) (Art. **2515.**) The specific resistance of a substance is the resistance of a cube of the substance 1 cm. on each side, measured between two parallel faces of the cube. The resistance of a piece of a substance of any length and sectional area being known, the specific resistance of that substance may be found from the formulas given, as follows:

The length of the piece in the example is 1 inch. Its resistance, if its length were 1 cm., its area being unchanged, may be found from formula **406,**

$$r_2 = \frac{r_1 \, l_2}{l_1}.$$

In this example $r_1 = .00000824$ ohm, $l_1 = 1$ in., $l_2 = .3937$ in. (1 cm.).

$$\text{Then, } r_2 = \frac{.00000824 \times .3937}{1} = .0000032441 \text{ ohm,}$$

the resistance of a body of this substance 1 cm. long and 1 sq. in. sectional area. To find the resistance if the area be reduced to 1 sq. cm. (.155 sq. in.), formula **407** may be used:

$$r_2 = \frac{r_1 \, a_1}{a_2}.$$

In this example, as it now stands, $r_1 = .0000032441$, $a_1 =$ 1 sq. in., $a_2 = .155$ sq. in.

Then, $r_2 = \dfrac{.0000032441 \times 1}{.155} = .00002093$ ohm. Ans.

Or, multiplying by 1,000,000, gives 20.93 microhms. Ans.

(1211) Formula **451**, $f = H \sin m°$, may be used in solving this example.

The magnet being short in proportion to the diameter of the coil, the value of f may be found from formula **448**,

$$f = \frac{2\pi A t}{r}.$$

In this example, $A = \dfrac{.362}{10} = .0362$, $t = 8$, $r = 12$, $2\pi =$ 6.2832.

Then, $$f = \frac{6.2832 \times .0362 \times 8}{12} = .15163.$$

As the instrument is set up in Chicago, the value of H (Table 84) is .184;

then, $$.15163 = .184 \times \sin m°,$$

or $$\sin m° = \frac{.15163}{.184} = .82408,$$

and, from the table, $m° = 55° \ 30'$, nearly. Ans.

(1212) The fundamental equation of the Wheatstone bridge (formula **459**) is

$$X = \frac{M}{N} \times P.$$

It will be seen from the connections of the battery and galvanometer circuits in the diagram that the coils lying between c and a form the upper balance arm of the bridge, and hence, in this example, $M = 1$ ohm; the coils between a and d form the lower balance arm, and hence $N = 100$ ohms; and the coils between d and b form the adjustable arm, and hence $P = 500 + 200 + 20 + 2 + 1 = 723$ ohms. Substituting these values in the fundamental equation,

$$X = \frac{M}{N} \times P = \frac{1}{100} \times 723 = 7.23 \text{ ohms. Ans.}$$

(1213) (Art. **2546.**) Formula **466,** $R = r\left(\frac{d}{d_1} - 1\right)$, applies to this example.

The source of the E. M. F. in this case being a dynamo giving a constant potential of 115 volts, the deflection of the voltmeter when connected directly to the dynamo will be 115; i. e., $d = 115$.

From the example, $d_1 = 17.5$ and $R = 18,000$.

Then, $$R = 18,000 \times \left(\frac{115}{17.5} - 1\right) =$$

$$18,000 \times 5.5714 = 100,285 \text{ ohms},$$

or, practically, .1 megohm. Ans.

(1214) The ratio of the drops through two conductors in series is the same as the ratio of their resistances. If at 15° C. their resistances are equal, at any higher temperature their resistances would both be increased by an amount depending on their temperature coefficient.

The temperature coefficient of copper is .00388, while that of mercury is .00088, so that the resistance of the copper at a temperature of 62° C., which is a rise of 47° C., would be, from formula **460,**

$$r_2 = r_1\,(1 + .00388\,C°),$$

where r_1 = resistance at 15° C. and r_2 = resistance at 15 + C.°,

or $$r_2 = r_1 \times [1 + (.00388 \times 47)] = 1.1824\,r_1,$$

and the resistance of the mercury would be

$$r_2 = r_1\,(1 + .00088\,C°),$$

or $$r_2 = r_1 \times [1 + (.00088 \times 47)] = 1.0413\,r_1.$$

The value of r_1 being the same for both the mercury and the copper part of the circuit, at 15° C., since the drop is the same, at 62° C. the drops will be directly proportional to the resistances, or

drop through mercury : drop through copper :: $1.0413\,r_1$: $1.1824\,r_1$, and their ratio would be $\frac{1.0413}{1.1824}$. Ans.

(1215) The current in the coil of a tangent galvanometer being directly proportional to the tangent of the angle of deflection (formula **450**), the shunt in this example must reduce the current in the galvanometer from $K \times \tan 58°$ to $K \times \tan 31°$; from the table, $\tan 58° = 1.6003$ and $\tan 31° = .6009$. Then, the shunt reduces the current from $1.6003\ K$ to $.6009\ K$. The multiplying power of the shunt, $n + 1$, may be obtained from formula **454**,

$$C_g = \frac{C}{n+1}.$$

In this example $C_g = .6009\ K$. The total current being the same as when the whole current was passing through the the galvanometer, $C = 1.6003\ K$.

Then, $$.6009\ K = \frac{1.6003\ K}{n+1},$$

or $$(n+1) \times .6009\ K = 1.6003\ K,$$

and $$n + 1 = \frac{1.6003\ K}{.6009\ K} = 2.6632.$$

Then, $$n = 1.6632.$$

Also, $$n = \frac{R_g}{R_s}, \text{ or } R_s = \frac{R_g}{n}.$$

From the example, $R_g = 400$ ohms.

Then, $$R_s = \frac{400}{1.6632} = 240.5 \text{ ohms. Ans.}$$

(1216) (Art. **2507.**) The combined resistance of A and X in parallel is to the resistance of B as 4.1 is to 5.6. B being 77 ohms, then the combined resistance of A and X in parallel is

$$\frac{4.1 \times 77}{5.6} = \frac{315.7}{5.6} = 56.375 \text{ ohms.}$$

(*b*) The current flowing in the circuit is $\frac{5.6}{77} = .07273$ ampere. Ans.

(*a*) Since through the resistance A of 196 ohms, enough of this current is passing to cause a drop of 4.1 volts, or $\frac{4.1}{196} = .02092$ ampere, the balance, $.07273 - .02092 = .05181$

ampere, is flowing through the unknown resistance, causing a drop of 4.1 volts. (See Arts. **2323** and **2324.**) The unknown resistance is, then,

$$R = \frac{E}{C} = \frac{4.1}{.05181} = 79.135 \text{ ohms.} \quad \text{Ans.}$$

(1217) The problem may be solved by first finding the value of $A\,t$ in formula **448,** the other quantities being given. The diameter of the coil $= 19\frac{11}{16}$ in. $= 19.6875 \times 2.54 = 50.006$ cm.

Then, in the formula $f = \dfrac{2\,\pi\,A\,t}{r}$,

$$f = .7,\ r = \frac{50.006}{2} = 25.003,\ 2\,\pi = 6.2832; \text{ hence,}$$

$$.7 = \frac{6.2832\,A\,t}{25.003}, \text{ or } .7 = .2513\,A\,t, \text{ and } A\,t = \frac{.7}{.2513} = 2.7855.$$

In this expression, A = C. G. S. amperes; so this value must be multiplied by 10 to give the correct result. $10 \times 2.7855 = 27.855$ = product of amperes and turns. Ans.

(1218) See Art. **2507.**

(*a*) The two resistances being directly proportional to the drops through them,

$$X : 41 :: 7.54 : 10.66,$$

or
$$X = \frac{41 \times 7.54}{10.66} = 29 \text{ ohms.} \quad \text{Ans.}$$

(*b*) The current may be found from Ohm's law (Art. **2310**). The known resistance being 41 ohms, and the drop through it 10.66 volts, the current is

$$C = \frac{E}{R} = \frac{10.66}{41} = .26 \text{ ampere.} \quad \text{Ans.}$$

(1219) See Art. **2458.**

(1220) (Art. **2462.**) By bringing to bear upon the poles of the magnet a known constant force in a different direction from that exerted by the coil and measuring the resultant of the two forces.

(1221) (Art. **2546.**) The total resistance of the rod may be found from Ohm's law (Art. **2310**),

$$R = \frac{E}{C}.$$

In this case, $C = 35.4$, $E = .00875$. Then, $R = \frac{.00875}{35.4} = .0002472$ ohm, the resistance of 25 inches of the rod. The resistance of 1 foot of the rod would then be $\frac{.0002472}{25} \times 12 = .00011866$ ohm. Ans.

(1222) (*b*) (Art. **2488.**) In formula **454**, $C_g = \frac{C}{n+1}$; there is given in this example C_g and C. C_g must be the same with or without the shunt, as the deflection is the same in both cases.

Then, $C_g = .062$ ampere,
$C = .558$ ampere.

Hence, $.062 = \frac{.558}{n+1}$, or $\frac{.558}{.062} = n + 1 = 9$, the multiplying power of the shunt. Ans.

(*a*) $n = 9 - 1 = 8$; also, $n = \frac{R_g}{R_s}$, or $R_s = \frac{R_g}{n} = \frac{1{,}086}{8} = 135.75$ ohms, resistance of the shunt. Ans.

(*c*) The combined resistance of the galvanometer and shunt may be found from formula **412**, Art. **2325.** The joint resistance is

$$R'' = \frac{r_1 r_2}{r_2 + r_1}.$$

In this case, $r_1 = 1{,}086$ ohms and $r_2 = 135.75$ ohms.

Then, $R'' = \frac{1{,}086 \times 135.75}{135.75 + 1{,}086} = \frac{147{,}424}{1{,}221.75} = 120.67$ ohms.

As the office of the compensating resistance is to add to the joint resistance of galvanometer and its shunt sufficiently to make the total resistance equal the original resistance of the galvanometer alone, then, in this case, the compensating resistance $= 1{,}086 - 120.67 = 965.33$ ohms. Ans.

(1223) (Arts. **2493** to **2495.**) 2 hours $= 60 \times 2 \times 60 = 7{,}200$ seconds. We may here use formula **455,** where the given loss in weight $w_1 - w_2$ is 2.1 grams. Then the strength of current is

$$C = \frac{2.1}{.00009324 \times 7{,}200} = 3.128 \text{ amperes. Ans.}$$

(1224) (Art. **2458.**) On the distance between the conductor and the magnet pole; on the length of the conductor; on the strength of the current; and on the strength of the magnet pole.

(1225) See Art. **2492** and Art. **2506.**

(*a*) The resistance may be found by Ohm's law.

$$R = \frac{E}{C}.$$

The answer should be carried to four significant figures, the accuracy of the instruments not being within 1%.

Then, $R = \frac{4.9}{2.3} = 2.13$ ohms. Ans.

(*b*) In this case, the result should be carried out to six significant figures, the accuracy of the instruments being within between .1% and .01%.

Then, $R = \frac{4.9}{2.3} = 2.13043$ ohms. Ans.

(1226) The scale, the beam of light, and the line from the zero of the scale to the mirror form a right-angled triangle, and the angular deflection of the beam of light from its zero position can be calculated from the length of the part of the triangle formed by the scale, and the distance between the center of the scale and the mirror, by applying the rule given in the section on Geometry and Trigonometry:

$$\text{the tangent of the angle} = \frac{\text{side opposite}}{\text{side adjacent}}.$$

Then, when the spot of light is deflected to the end of the scale, the "side opposite" is equal to one-half the total length of the scale, or 24 inches. The side adjacent is the

distance from the mirror to the center of the scale, or 30 inches ($2\frac{1}{2}$ ft.).

Then, $\frac{24}{30} = .8$, the tangent of the angle made by the beam of light, when deflected, with its zero position.

From the table, $.8 = \tan 38° 40'$, nearly. Now, as the angle which the beam of light makes with its zero position is twice the angle of deflection of the needle (Art. **2474**), the deflection of the needle is $\frac{38° 40'}{2} = 19° 20'$.

Similarly, when the deflection of the ray of light extends over 12″ of the scale, the tangent of the angle is $\frac{12}{30} = .4$, the angle is $21° 48'$, and the angle of deflection of the needle is $\frac{21° 48'}{2} = 10° 54'$; and when the deflection of the ray of light is 6 inches, the tangent of the angle is $\frac{6}{30} = .2$; the angle is $11° 18'$, and the angle of deflection of the needle is $\frac{11° 18'}{2} = 5° 39'$.

To find the current required to cause these deflections, formulas **449** and **448** should be used. In formula **449**,

$$f = H \tan m°,$$
$$H = .184;\ \tan m° =$$
$$(a)\ \tan 19° 20' = .35085,$$
$$(b)\ \tan 10° 54' = .19257,$$
$$(c)\ \tan 5° 39' = .09893.$$

Then,
$$(a)\ f = .184 \times .35085 = .064556,$$
$$(b)\ f = .184 \times .19257 = .035433,$$
$$(c)\ f = .184 \times .09893 = .018203.$$

In formula **448**,

$$f = \frac{2\pi A t}{r},$$

f is as just given; $t = 6$; $r = 12$ cm.

Then (a), $$.064556 = \frac{6.2832 \times A \times 6}{12} = 3.1416\,A,$$

or $$A = \frac{.064556}{3.1416} = .02055.$$

Multiplying by 10, to get the amperes, gives .2055. **Ans.**

$$(b)\quad .035433 = \frac{6.2832 \times A \times 6}{12} = 3.1416\, A,$$

or

$$A = \frac{.035433}{3.1416} = .01128.$$

Multiplying by 10, to get the amperes, gives .1128. Ans.

$$(c)\quad .018203 = \frac{6.2832 \times A \times 6}{12} = 3.1416\, A,$$

or

$$A = \frac{.018203}{3.1416} = .005794.$$

Multiplying by 10, to get the amperes, gives .05794. Ans.

(1227) See Art. **2522.**

(*a*) Formula **464,** for finding the insulation resistance of a line by this method, is

$$I = \frac{R \tan d}{\tan d_1},$$

where R = value of the known resistance, d = angle of deflection when R is in circuit, and d_1 = angle of deflection when the battery is connected to the earth.

In this case, $R = 15{,}000$ ohms, $d = 69°$, $d_1 = 30°\ 30'$.

Then, $\tan d = 2.605$ and $\tan d_1 = .5891$, and

$$I = \frac{15{,}000 \times 2.605}{.5891} = \frac{39.075}{.5891} = 66{,}330 \text{ ohms} =$$
$$\frac{66{,}330}{1{,}000{,}000} = .06633 \text{ megohm.} \quad \text{Ans.}$$

(*b*) (Art. **2522.**) In this case the length of the line is given as 10 miles. The insulation resistance per mile is, then, $10 \times .06633 = .6633$ megohm. Ans.

(*c*) There being 440 poles, the insulation resistance per pole is $440 \times .06633 = 29.2$ megohms. Ans.

NOTE.—This method not being accurate within 1%, four significant figures are sufficient in any calculation or in the result.

(1228) See Art. **2477.**

(1229) (*a*) The diagram should be about like Fig. 100. (See Arts. **2524** and **2526.**)

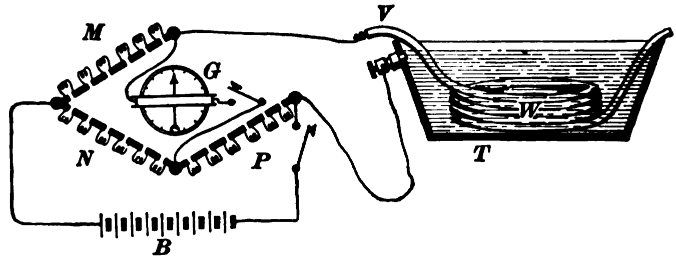

FIG. 100.

(*b*) The diagram should be about like Fig. 101. (See Art. **2546.**)

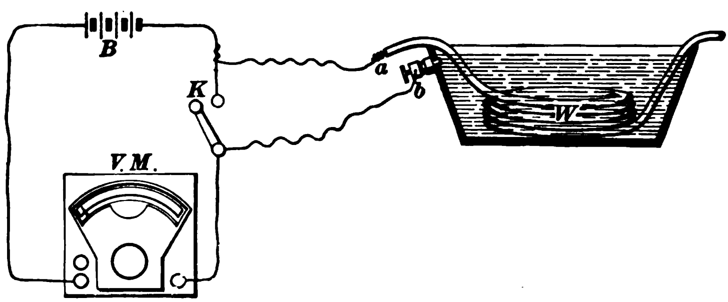

FIG. 101.

BATTERIES.

(QUESTIONS 1230–1292.)

(1230) See Art. **2550.**

(1231) (*a*) See Art. **2570.**
(*b*) See Art. **2568.**
(*c*) See Art. **2569.**

(1232) (*a*) The cause of the E. M. F. of a cell is the affinity which exists between elements of the various substances which make up the cell.

(*b*) The current from a cell is maintained by the combination of various elements of the substances constituting the cell keeping up the E. M. F., which maintains the current. (See Art. **2576.**)

(1233) See Arts. **2599** and **2654.**

(1234) The gravity Daniell cell employs as an anode, zinc; as an electrolyte, a solution of zinc sulphate; as a depolarizer, a solution of copper sulphate; as a cathode, copper. The liquids have different specific gravities, so that they may be kept one over the other in the cell without mixing. The cell gives a constant E. M. F. of about 1.07 volts. The zinc sulphate solution is continually formed while the cell is in action, and must be occasionally removed; the copper sulphate solution is continually reduced, and must be occasionally replaced. The cells are used largely in telegraph and fire-alarm work. (See Arts. **2655** to **2671** and **2706** to **2709,** inclusive.)

(1235) This may be calculated from the electrochemical equivalent as follows: 1 ampere-hour = 1 ampere for

1 hour, or $60 \times 60 \times 1 = 3,600$ coulombs. As the consumption of zinc is .0003382 gram per coulomb (Table 89, column 6), the consumption per ampere-hour is $3,600 \times .0003382 =$ 1.218 grams per ampere-hour. (See Art. **2718.**)

(1236) The maximum current will flow when the internal resistance equals the external. (Art. **2722.**) In this case, the external resistance is 3 ohms, and the combination of cells which will give this resistance will be found by calculation to be 2 sets in parallel, with 6 cells in series in each set. The internal resistance of a set of 6 cells in series, of 1 ohm resistance each, would be 6 ohms, and the resistance of two such sets would be $\frac{6}{2}$ or 3 ohms. (Art. **2721.**) The E. M. F. of a battery thus arranged would be $6 \times 2 = 12$ volts (Art. **2721**), and the resistance of the circuit is 3 ohms internal + 3 ohms external, or 6 ohms; the current would then be $\frac{12}{6} = 2$ amperes. Ans.

(1237) Hg_2O would be called mercur*ous* oxide, and $Hg\,O$ would be called mercur*ic* oxide. (See Art. **2571.**)

(1238) See Art. **2583.**

(1239) (*a*) The formation of a layer of hydrogen on the cathode of a cell is called polarization.

(*b*) The removal of such a layer by mechanical or chemical means is called depolarization. (Art. **2594.**)

(1240) (*a*) The most important applications of primary batteries are to the telegraph, telephone, signal service, cautery purposes, electroplating, and for running small motors. (See Arts. **2705** to **2716,** inclusive.)

(*b*) See Arts. **2807** to **2815.**

(1241) The positive plates. (Art. **2740.**)

(1242) The battery output is 6,000 ampere-hours, during a day of 24 hours. The total output of the plant is 2,000 amperes multiplied by 24, or 48,000 ampere-hours. The dynamo plant furnishes direct to the line $48,000 - 6,000 =$ 42,000 ampere-hours. The dynamo plant furnishes to the line, *through the battery*, $6,000 \div .80 = 7,500$ ampere-hours.

(*a*) The total daily output of the generating plant is the sum of the direct output to the line and the battery output to the line, or $42,000 + 7,500 = 49,500$ ampere-hours. Ans.

(*b*) The dynamo plant running twenty-four hours at full capacity must furnish $49,500 \div 24 = 2,062.5$ amperes. Ans.

(1243) See Art. **2554.**

(1244) See Art. **2590.**

(1245) Any cell which answers to the description given in Arts. **2607** to **2621** will do. For example, the zinc, ammonium chloride, and carbon cells, of which the Little Giant, Law cell, etc., are examples.

(1246) (*a*) Because the two solutions will mix and cause local action if the cell is "open-circuited." (Art. **2661.**)

(*b*) Because the cell will polarize if called upon to deliver a current for any considerable time. If used for intermittent currents, the hydrogen layer on the cathode will pass off in the periods of rest of the cell. (Arts. **2594** and **2602.**)

(1247) The formation of a layer of the insoluble white sulphate of lead on the positive plates. (Art. **2736.**)

(1248) The rapid formation of insoluble sulphate in the plugs of active material causes them to expand and thus distort the inelastic lead grid. (See Art. **2741.**)

(1249) The great weight of accumulators necessary; the rapid disintegration of the plates; and the expense of handling the batteries. (See Arts. **2802, 2803,** and **2804.**)

(1250) The E. M. F. of the cell is reduced. (See Art. **2590.**)

(1251) (*a*) Because the cost of the materials consumed makes the cost of the energy many times greater than that obtained from dynamo machinery. (See Art. **2601.**)

(*b*) Because in many instances the cost of the energy is unimportant compared with their constancy and the little attention required. (See Art. **2705.**)

(1252) (*a*) See Art. **2734.**

(*b*) It indicates that the cell is completely charged. (See Art. **2734.**)

(1253) (*a*) The number of cells required is found by dividing the required E. M. F. by the average E. M. F. of each cell (Art. **2822**), or $\frac{52}{1.9} = 28$ cells. The size of each cell is found from the number of ampere-hours required for the devices which consume the current during one charge. (Art. **2825.**) In this case the cells are to be charged once every 7 days, and of the 32 lamps installed, 2 burn for 10 hours per day, and will then use $2 \times 10 \times 7 = 140$ ampere-hours in the week. Of the balance, 6 burn for 5 hours per day, and will then use $6 \times 5 \times 7 = 210$ ampere-hours; the balance, $32 - 8 = 24$, burn for 1 hour per day, and will use $24 \times 1 \times 7 = 168$ ampere-hours. The capacity of the cells must then be $168 + 210 + 140 = 518$ ampere-hours. The plant would then consist of 28 cells, each of a capacity of 518 ampere-hours.

(*b*) Placed in two tiers, 14 in each tier, the floor space occupied would be that of 14 cells, each cell occupying $\frac{518}{3} = 172\frac{2}{3}$ sq. in. (Art. **2820.**) Then, $14 \times 172\frac{2}{3} = 2{,}417\frac{1}{3}$ sq. in., or 17 sq. ft., practically.

(1254) See Art. **2563.**

(1255) (*a*) Mechanical and chemical depolarization. (See Art. **2595.**)

(*b*) The chemical method. (See Arts. **2596** and **2597.**)

(*c*) Because mechanical depolarizers require some sort of mechanism to perform the required service, which therefore needs attention and is expensive. (See Art. **2596.**)

(1256) Art. **2633.** (*a*) Cells with a liquid depolarizer.

(*b*) See Art. **2604.**

(1257) See Arts. **2740, 2743,** and **2746.**

(1258) See Art. **2742.**

(1259) (*a*) A chemical action is the combination of two or more substances forming another substance or substances of different physical properties, or is the decomposition of some substance with a similar result. Such action is a manifestation of one form of energy.

(*b*) The energy of such actions usually appears in the form of heat.

(*c*) The amount of heat liberated by the formation (or absorbed by the decomposition) of unit weight of a substance is called the heat of formation of that substance. (See Art. **2559.**)

(1260) (*a*) and (*b*) See Arts. **2602** to **2606.**

(1261) See Art. **2663.**

(1262) (*a*) The active material on the negative plate is reduced to spongy lead, sulphuric acid being formed; the positive plate is oxidized, forming lead peroxide. (See Art. **2733.**)

(*b*) The formation of lead oxide and then sulphate by the action of the acid on the negative plate, and the reduction of the peroxide on the positive plate to lead oxide, or to spongy lead. The percentage of acid in the electrolyte is reduced. (See Art. **2735.**)

(1263) See Art. **2752.**

(1264) (*a*) The symbol of an element is the first letter, or the first letter and one other, of its English or Latin name. (Art. **2557.**)

(*b*) Each element has assigned to it a symbol in order that the element may be referred to without constant repetition of its entire name. (Arts. **2557** and **2562.**)

(*c*) The chemical formula of a substance is an expression of the various elements which enter into a substance and the relative amounts of each element in that substance. The presence of the elements is indicated by writing the symbol of each element, and the proportion of each element in the substance is shown by a small number suffixed to the symbol of the element. (Art. **2562.**)

(**1265**) Bichromate cells of the second class are liable to local action, by which the zinc is consumed, if the cells are left with the external circuit open. (Art. **2623.**)

(**1266**) The Faure process consists of coating the surface of a lead plate with a layer of oxide or sulphate of lead, such that on the passage of a charging current this coating is converted into the proper active material for use in accumulators. (Art. **2730.**)

(**1267**) Because at a high rate of discharge all the active material is quickly converted on its surface, and as the acid can only penetrate slowly into the body of the active material, the E. M. F. falls to its limiting value before all the active material is consumed. (Art. **2745.**)

(**1268**) Long-continued overcharging will remove deposits of the white insoluble sulphate from the positive plate. (See Art. **2754.**)

(**1269**) (*a*) The atomic weight of an element is the relative weight of an atom of that substance as compared with an atom of the lightest substance (hydrogen). (See Art. **2561,** and column 3, Table 89.)

(*b*) The chemical equivalent of an element is the weight of that element which will replace unit weight of hydrogen in any substance, or will combine with unit weight of hydrogen. (See Art. **2564,** and column 5, Table 89.)

(*c*) The valency of an element is the ratio of its atomic weight to its chemical equivalent. (See Art. **2565,** and column 4, Table 89.)

(*d*) The atomicity of an element is the same as its valency. (See answer to (*c*).)

(1270) (*a*) See Art. **2621** and Table 92.
(*b*) See Art. **2609** and Table 92.
(*c*) See Art. **2617.**

(1271) Potassium zincate is a compound oxide of potassium and zinc. (See Art. **2683.**)

(1272) See Art. **2699.**

(1273) See Art. **2728.**

(1274) Because the acid will slowly penetrate into the internal portions of the active material, and the original E. M. F. will be restored. (See Art. **2744.**)

(1275) (*a*) They serve to equalize the load on the generators and engines, thus allowing the machinery to be operated at its maximum efficiency. (See Art. **2792.**)

(*b*) See Art. **2793.**

(1276) (*a*) and (*b*) The valency of oxygen is II, while that of iodine is I. Consequently, 2 atoms of iodine will replace 1 atom of oxygen (Art. **2565**), and 1 molecule of $H_2 O$ will become 2 molecules of $H I$.

(1277) Because the free elements all through the liquid recombine as fast as liberated, except at the electrodes, where they either combine with the substances there or appear in the free state. (See Art. **2592.**)

(1278) A standard cell should keep its E. M. F. constant at a certain value throughout the life of the materials of the cell, and should be affected as little as possible by heat, etc. (See Arts. **2693** to **2696,** inclusive.)

(1279) See Art. **2727.**

(1280) The output in ampere-hours on discharge divided by the input in ampere-hours required to charge the cell is its ampere-hour efficiency. (See Art. **2749.**)

(1281) (*a*) See Art. **2778.**
(*b*) See Arts. **2779** and **2782.**

(1282) (*a*) Light weight for a given output, and freedom from buckling or sulphating. (See Arts. **2786** and **2788.**)

(*b*) Its low E. M. F., care and special operation required in charging and discharging, and liability to local action. (See Arts. **2788** and **2789.**)

(1283) (*a*) See Art. **2555.**
(*b*) See Art. **2556.**
(*c*) and (*d*) See Art. **2567.**

(1284) (*a*) As the number of atoms of any two combining elements which make up a molecule are inversely proportional to the valencies of those elements (Art. **2565**), in this case the valency of the copper in the $Cu\,Cl$ solution must be I, and in the $Cu\,Cl_2$ solution must be II. The amount of copper which will be deposited per coulomb being its electrochemical equivalent, then from the $Cu\,Cl$ solution will be deposited .0006568 gram per coulomb, while from the $Cu\,Cl_2$ solution will be deposited .0003284 gram per coulomb. (Column 6, Table 89.) In this case the current is 4 amperes, maintained for $10 \times 60 = 600$ seconds, and is thus equal to $4 \times 600 = 2{,}400$ coulombs. Then, from the first bath ($Cu\,Cl$) will be deposited $2{,}400 \times .0006568 = 1.576$ grams of copper.

(*b*) From the second bath ($Cu\,Cl_2$) will be deposited $2{,}400 \times .0003284 = .7882$ gram of copper.

(*c*) This is answered above. (See Art. **2571.**)

(1285) The bichromate cell is the best example. (See Arts. **2622** to **2631**, inclusive.)

(1286) See Arts. **2689** and **2725.**

(1287) The plate at which the current *enters* on charging and leaves on discharging. (Art. **2726.**)

(1288) The Lalande-Chaperon and Edison-Lalande. (Art. **2783.**)

(1289) The materials used are as follows: Anode, zinc; electrolyte, solution of caustic potash (potassium hydrate;)

depolarizer and cathode, oxide of copper. The action is the oxidation of the zinc and its formation with the potassium hydrate of potassium zincate; the depolarization is effected by the combination of the evolved hydrogen with the oxygen of the copper oxide. (See Art. **2684.**)

(1290) Oxygen. (See Art. **2597.**)

(1291) From the rule given in Art. **2724,** the current in the external circuit $= \frac{E}{R} = \frac{1.6}{4.5} = .3555$ ampere. The drop in the cell is $1.9 - 1.6 = .3$ volt, and the resistance of the cell $= \frac{E}{C} = \frac{.3}{.3555} = .844$ ohm. Ans.

(1292) An accumulator stores energy in the form of potential chemical energy. (See Art. **2748.**)

ELEMENTS OF TELEGRAPH OPERATING.

(1) The key is a device for opening and closing an electric telegraph circuit; or it may be defined as an instrument used in telegraphy, by the manipulation of which electric impulses representing dots and dashes are made to apparently travel from a sending to a receiving office.

(2) The essential parts of a telegraph key are the base, lever, circuit-closer, spring, adjustable trunnion screws with locknuts, a screw with a locknut for regulating the play of the lever, two contact points usually made of platinum; and on a separately mounted key, either two binding posts or two legs.

(3) The Morse alphabet should be so well known that the combination of dots, dashes, and spaces representing any letter, numeral, or punctuation mark in common use can be given instantly without any hesitation and so well that any of the characters when properly made on the sounder can be recognized immediately without having to think about it even for an instant.

(4) *a* and *n*, *b* and *v*, *c* and *r*, *d* and *u*, *g* and *w*, *q* and *x*, *z* and *&*, *4* and *8*.

(5) See Arts. **6** and **7**.

(6) (*a*) These intervals are called breaks.

(*b*) As short as possible, but all breaks should be alike, not some short and others long.

(*c*) A break is generally considered to be equivalent in duration to a dot. See Art. **8.**

(7) It consists of a sounder and key mounted upon the same base, a battery, and some connecting wire.

(8) The space between every three figures is generally made larger than the space between the other figures, thus separating them into groups of three by larger spaces where in writing commas would be used.

(9) A sounder is an electromagnetic device so constructed that it gives forth a click or sound every time the electric circuit is opened or closed by a telegraph key connected in the same circuit with it.

(10) The essential parts of a sounder are the two soft-iron cores upon which are wound two coils of insulated copper wire, a soft-iron armature fastened to a brass (or aluminum) lever, two trunnion screws with locknuts, a spring, a metal piece called an anvil, two screws with locknuts for limiting the play of the armature lever, and a screw for regulating the tension of the spring.

(11) (*a*) Open and closed circuit cells.
(*b*) See Art. **19.**

(12) See Fig. 6.

(13) (*a*) The armature of a sounder should have a play of about $\frac{1}{16}$ inch.

(*b*) There should always be room enough to pass one thickness of ordinary writing paper between the iron armature and the cores.

(*c*) Because the iron armature would stick to the iron core and would not move away promptly when the electric current ceased to flow through the coils.

(14) (*a*) The key contact points should be kept clean.

(*b*) All binding posts should be kept tight.

(15) By sending the letter representing the office desired three or more times, immediately followed by the letters representing the calling or home office. This call is repeated until the desired office answers.

(16) By sending the letter *I* several times, immediately followed by the letters representing the office that is making the reply.

(17) See Art. **66.**

(18) Because the earth acts as a conductor and may, therefore, be used in the place of one line wire.

(19) See Arts. **43** and **44.**

(20) See Art. **67.**

(21) (*a*) The date consists of the name of the place (town and state usually), month, day of the month, and the year.

(*b*) The operator does not send the month and year.

(*c*) The operator always sends the abbreviation *fm* or *fr*, signifying "from," before the date.

(22) Beginners are apt to make the final dot or dash too long, instead of making the last one the same as those preceding.

(23) (*a*) The number of the street in the address should be transmitted in numerals, but the name of a street when a number, as Fourth street, should be transmitted in words.

(*b*) The word *to* is always sent before and a period (.) after the address.

(24) (*a*) The body of the message follows the period that closes the address.

(*b*) The abbreviation *sig* for "signature" follows the body of the message.

(*c*) The signature or the name of the party or parties sending the telegram follows the abbreviation *sig*.

(25) The difficulty consists in making the correct number of dots without apparently counting them and without prolonging the last dot into a dash.

(26) By telegraphing back the letters *O K* followed by his own private letters or mark. See Art. **75.**

(27) Because it is easy by improperly proportioning the elements and spaces to transform a letter *k* into a letter *d*, or into *nt* or *ta*.

(28) *Hr city* or *hr tru*, *no*, *ck*, *fm* or *fr* (for "from") before the date, *to* before the address, and *sig* before the signature.

(29) *i* becomes *a*, *s* becomes *u*, *h* becomes *v*, and *p* becomes *4*.

(30) He breaks; that is, he opens his circuit-closer and telegraphs back the letters *ga* (go ahead), giving immediately the last word correctly received. See Art. **79.**

(31) (*a*) The sending operator, commencing at the period, transmits the first letter in each word until the missing portion is located. See Arts. **76** and **77.**

(*b*) He makes seven or more dots, transmits the abbrevition *msk* (mistake), and begins again with the last word he made correctly.

(32) The so-called spaced letters are *c*, *o*, *r*, *y*, *z*, and *&*.

(33) The ability to send accurately and steadily without mistakes is more desirable than merely high speed.

(34) (*a*) It is a system of abbreviations consisting of single letters and combinations of two or more letters that arbitrarily represent figures, words, and whole phrases.

(*b*) It is extensively used by the press associations for telegraphing newspaper matter.

(35) (*a*) Because, by making the space in the spaced letter *c* too short, *c* becomes *s*.

(*b*) Because, by making the space in the spaced letter *o* too long, *o* becomes *ee*, and conversely by making the space between *e* and *e* too short, *ee* becomes *o*.

(*c*) Because, by making the space in the spaced letter *r* too short, *r* becomes *s*.

(*d*) Because, by making the space in the spaced letter *z* too short, *z* becomes *h*.

(*e*) Because, by making the long dash representing *l* too short, *l* becomes *t*.

(*f*) Because, by making the space between *m* and *e* too short, the *me* becomes *g*.

(*g*) Because, by making the space after the dot representing *e* a third of a unit too short and the next space (the space in the spaced letter *r*) a third of a unit too long, *er* becomes *oi* (*e* · *r* · ··   *o* · · *i* ··)

(36) *At* becomes *w*, *ed* becomes *x*, *we* becomes *1*, *ve* becomes *3*, *ui* becomes *2*, and *ud* becomes a . (period).

(37) Instead of the characters intended, the following letters may be incorrectly transformed; as *j* into *nn* or *ke*; *k* into *d*, *nt*, or *ta*; *th* into *di*, *do*, *nr*, *ns*, *ty*, *tz*, *t&*, or *8*; *an* into *b*, *h*, *j*, *q*, *v*, *x*, *y*, *z*, *&*, *ai*, *ao*, *in*, *on*, *ed*, *fe*, *eg*, or *we*; *i* into *a*, *n*, or *o*; *s* into *c*, *d*, *f*, *g*, *k*, *r*, *u*, *w*, *ae*, *it*, *ot*, *ea*, *eo*, or *en*; *h* into *b*, *j*, *q*, *v*, *x*, *y*, *z*, *&*, *1*, *7*, *9*, or , ; *p* into *2*, *3*, *4*, *8*, *os*, *is*, *ir*, *or*, *he*, *ey*, or *ez*.

(38) Because by making the space between the two letters in each case too short (one instead of three units in length), the single letter or the figure following the two letters are made instead of the two letters; and conversely, by making the spaces in the single letters or in the figure too long, the preceding two letters are made instead of the single letter or figure.

TELEGRAPHY.

(PART 1.)

---

(1) (*a*) Electric telegraphy is the art, science, or process of transmitting intelligible signals or signs between distant points by means of electric impulses apparently moving between those points.

(*b*) Visible and audible signals may be used.

(*c*) The essential parts are the transmitting and receiving apparatus and the line wire, or its equivalent in wireless telegraphy.

(2) Because it is not practical to obtain in the long line circuit a current strong enough to operate a sounder, but a current strong enough to operate a relay can be readily obtained; and the relay is able to open and close a local circuit containing a sounder and a battery that can furnish sufficient current to work the sounder. See Arts. **24, 96,** and **99.**

(3) (*a*) In 1832.

(*b*) In 1837.

(*c*) Alfred Vail.

(4) Connect the cells in such a series-parallel combination that the internal resistance of the whole battery will equal the external resistance.

(5) (*a*) As many as 30 or even 40 stations are often connected in one line circuit.

(*b*) Since only one message can be sent at a time on lines not arranged for the simultaneous transmission of two or more messages, only one operator can be sending at one time, and this message may be of interest to only one or a few stations out of the whole 30 or 40. Consequently, all the other stations have to remain idle until the one sending gets through, or else interrupt him if their business is so very much more important that it gives them the right to do so.

(6) (*a*) The electromagnet.

(*b*) Independently by both Sturgeon and Henry.

(7) (*a*) A relay serves the purpose of receiving feeble currents from a line and causing them, by the movement of a light armature, to open and close a local circuit containing a battery and local instrument, from which the message may be read by sound or sight.

(*b*) The principal parts of a relay are an electromagnet having a light armature, suitable contact points, and retractile spring.

(8) Put pegs into the holes *22*, *5*, *13*, and *7*, or *23*, *4*, *13*, and *7*.

(9) Since the resistance of all 10 relays is to be made equal to the sum of the resistance of the line wire and the internal resistance of the cells, it follows that the internal resistance of all the cells must be equal to 1,500 − 1,230 = 270. Each cell has an internal resistance of 3 ohms; hence, the total number of cells required is 270 ÷ 3 = 90 cells. Ans.

From formula **11**,

$$C = \frac{s\,e}{\dfrac{s\,b}{p} + r + l}.$$

Substituting the values given in the example,

$$C = \frac{90 \times 1}{(90 \times 3) + 1{,}500 + 1{,}230} = .03 \text{ ampere.} \quad \text{Ans.}$$

(10) (*a*) Steinheil in 1837, and Morse in 1844.

(*b*) Because it saves running another wire for the return. Also, although the earth is a comparatively poor conductor, its practically unlimited area renders its resistance negligible in comparison with that of a long line wire. Hence the resistance of a long circuit will be less if the earth is used as a return path in place of one line wire. Moreover, since only one-half of the amount of wire is used, the expense of construction and maintenance is only about one-half of what it would be if a metallic return were used.

(11) Relays, embossing registers, and keys.

(12) See Art. **219.**

(13) (*a*) The needle telegraph system is one in which combinations of right and left deflections of a single vertical needle in front of a dial furnishes a system of signals representing the letters of the alphabet.

(*b*) On many railroad lines in Great Britain.

(14) These switches must both be kept closed at all times except at the one key where an operator is sending

(15) Ground, dynamo, lamp L', disk *4* in the first vertical row, strap *a*, spring jack *A*, desk set *U*, back to spring jack *A*, and out to the line.

(16) The Continental or Universal code is used all over Europe and practically all over the world on submarine cables. The Morse alphabet and numerals and the Phillips punctuation code is used all over the United States and Canada, except on submarine cables, on which the Continental is used.

(17) (*a*) On the principle that when a current flows (while a dot or dash is being sent) from a metal point through a paper moistened with certain chemicals (in this case a solution of potassium iodide) to a revolving metal drum, a permanent colored line is made upon the paper as it is drawn under the metal point.

(*b*) On the principle that the current when flowing from the revolving drum through the paper to the metal point makes the paper more slippery; that is, it reduces the friction between the point and the paper to such an extent as to allow a spring to move the lever against a stop and so close the local circuit. This local circuit is immediately opened again when the current stops, on account of the immediate increase in the friction to its normal value.

(*c*) It is very sensitive, requiring very little current.

(18) The simplest way would be to insert the wedge of the desk set in spring jack R and a plug in hole *r–4*. But this should *not be done if there is a plug in q–4, as shown*, because this would connect two lines through the same disk and lamp to the dynamo, which must be avoided, if possible. In this case this can easily be avoided by inserting the wedge of a desk set in the spring jack R and putting plugs in the holes *r–9*, *s–9*, and *s–4*, instead of only one plug in *r–4*. This may also be accomplished by inserting three plugs in various other positions.

(19) (*a*) According to the principles explained in Arts. **104** and **105**, we have, by formula **3**, $\frac{150}{300} = \frac{d'^4}{(.01)^4}$, in which .01 is the diameter of the No. 30 B. & S. wire used on a 150-ohm relay, as given in Art. **114**. From this, $d' = .0084$. This lies between sizes 31 and 32, hence use No. 32 B. & S.

(*b*) Since the ampere-turns must be kept constant, then, by formula **6**, $\frac{(C')^2}{(18)^2} = \frac{150}{300}$, from which $C' = 12.7$ milliamperes.

(20) The same as Fig. 10 (*b*), except that there will be three cells at each office, the three cells at the intermediate offices A and B being connected either between the line and relay or between the key and the line. See that the batteries at all four offices are in series with one another, that is, no battery opposing the others.

(21) (*a*) Because the Continental code has no spaced letters and for this reason fewer errors are made.

(*b*) The Morse code is about 5 per cent. more rapid than the Continental.

(22) (*a*) The Morse closed-circuit system is one in which all line batteries are in series in the line circuit, which is normally closed; thus current is normally flowing through the whole circuit, even though no message is being sent.

(*b*) Because, normally, even when no messages are being transmitted, the circuit is closed.

(23) The essential parts of an ordinary electromagnet are the iron cores, yoke, and armature, and the coils of insulated wire wound around the cores.

(24) (*a*) The combined resistance of all relays equal to the combined resistance of the line and batteries is commonly accepted as the proper relation, but it does not necessarily give the best arrangement.

(*b*) The resistance of all relays in the same circuit should be equal.

(25) (*a*) Yes.

(*b*) Not necessarily.

(*c*) All batteries in the line circuit must be in series with one another.

(26) By formula **7**,

$$d_w = .0288\sqrt[4]{\frac{1.1\,(.9^2 - .36^2)}{20}} = .01267.$$

From Table 1 the proper size of wire to use will be found to be No. 28 B. & S.

(27) (*a*) The Morse open-circuit system is the one in which the batteries are so arranged as to be cut out of the circuit except when signals are being sent. A battery is

required at each office. It is so named because all batteries are normally on open circuit, current flowing over the line only when signals are being transmitted.

(*b*) Yes.

(28) See Art. **58.**

(29) (*a*) The resistance of the coils will vary approximately as the square of the number of turns.

(*b*) The number of turns will vary approximately inversely as the square of the diameter over the wire and its insulation.

(30) See Art. **66.**

(31) (*a*) Anything that will decrease the reluctance, that is, the magnetic resistance, will increase the number of lines of force and therefore the pull. Hence, the strength of an electromagnet may be increased by increasing the cross-section of the iron parts, and by decreasing the length of the magnetic circuit, especially the air-gap part of the circuit.

(*b*) To make an electromagnet quick-acting, it is desirable to make the magnetic circuit short and the cross-section of the iron parts no larger than is really necessary.

(32) The sound may be concentrated in one direction by enclosing the sounder in a special box called a *resonator*.

(33) (*a*) All the batteries may be located at one office or subdivided among any number of offices. The current is more uniform and steady than in the open-circuit system.

(*b*) The closed-circuit system is not suitable for use with submarine cables, and there is a continual consumption of battery material even when no messages are being transmitted. See Arts. **33** and **34.**

(34) One way would be to disconnect the relay from the line and connect it directly to the terminals of a local battery, or, if necessary, to a stronger battery. If no signals can then be produced on the relay armature, the trouble is probably in the relay, although this does not prove that the

sounder circuit is all right. To test the sounder, connect the local-battery terminals direct to the sounder; that is, cut out the relay contacts. The sounder should close if it is not at fault and is properly adjusted. See, also, Arts. **76, 77,** and **187.**

(35) Put pegs in holes *20*, *9*, *10*, and *17*, or in *20*, *9*, *11*, and *16*, or in *21*, *8*, *11*, and *16*, or in *21*, *8*, *10*, and *17*.

(36) See Art. **163.**

(37) It is expressed by the ratio $\frac{L}{R}$. It is the time required for the current in a given circuit having an inductance L and a resistance R to reach $\frac{63}{100}$, or, approximately, $\frac{2}{3}$ of its final value. See Art. **91.**

(38) (*a*) There is a consumption of battery material only when messages are being sent, and this system is suitable for use on submarine cables.

(*b*) It requires at each office enough cells to work the whole line. Since the batteries at each office may not be in the same condition, the current may have a different strength for each office that sends, and, consequently, the relays may need to be readjusted whenever another office begins to send.

(39) (*a*) Place one plug in the hole between the plates p and z.

(*b*) Place one plug in the hole between plates p and z and another plug between plates p and y.

(40) (*a*) Quickness in action and then efficiency.

(*b*) The outside diameter of the coils should be three times the diameter of the core, the length of each coil should be equal to its diameter, the area of the poles the same size as the cores (that is, the ends of the cores neither reduced nor enlarged), cores only long enough to hold the necessary ampere-turns, armature broad, thin, and smaller in cross-section than the cores, yoke about the same size in cross-section as the cores.

(41) Leakage throws the relays out of adjustment, requiring their readjustment, and renders them heavier, as it is termed, that is, the current flowing through them is greater although the effective current is generally smaller than during dry weather. See Art. **65.**

(42) See Art. **67.**

(43) (*a*) To put the sounder in a local instead of in the line circuit is more economical and gives better service. To get the necessary current to operate the sounder in a long line circuit will require too high an electromotive force. On account of a sounder requiring so many more ampere-turns than a relay, it is much more practical and economical to use relays in the line circuit.

(*b*) Because it is as easy, if not easier, to read by sound as by sight. Furthermore, when reading by sound, the receiving operator's eyes are free to help him in writing down the message. Hence it is more difficult to copy from the register than from a sounder, especially when receiving with a typewriter, and, furthermore, the register is not so easily kept in adjustment, and not so quick as the sounder for commercial work, on account of the longer time it usually requires to get repetitions.

(44) Because the resistance can be determined so much easier than the number of turns. See Art. **97.**

(45) (*a*) Place a plug in hole *7*.
(*b*) Place plugs in holes *2* and *6*, or in holes *5* and *3*.
(*c*) Place a plug in hole *1*.

(46) (*a*) See Art. **75.**
(*b*) See Art. **75.**

(47) $$C = \frac{12 \times 1}{\frac{12 \times 3}{2} + 300 + 282} = .02 \text{ ampere.}$$

(48) Put plugs in holes *4*, *5*, and *3*, or in holes *4*, *6*, and *2*. Then if the west line is open, other things being all

right, the relay will not respond to signals made on the key. The local set should be overhauled or tested to prove that it is all right. See Art. **186.**

(49) The resistance of the battery is called the *internal* resistance, and all resistance in the circuit outside of, or external to, the battery is called the *external* resistance.

(50) Ground the east and west lines and connect the local battery to the relay and key leads; then if no current flows through the relay, evidently the trouble is in the office connections; if current does flow through the relay, then the trouble is in the line. See Arts. **187** and **188.**

(51) (*a*) In series.
(*b*) In series with the other cells.

(52) (*a*) Briefly, it is an arrester by which the lightning discharge is led to the ground. It usually consists of two electrical conductors or metallic plates separated by a thin insulating material, such as air, mica, or paraffined paper. One plate is connected to the line, the other to the ground. The static charge, due to the great difference of potential between the earth and the clouds that charge the line wire, breaks through this thin insulating medium and so escapes to earth in preference to going through the wire coils, which, on account of self-induction and the high frequency of the lightning-discharge current, offer a very great apparent resistance or impedance to the flow of such a current. The charge takes the path to earth through the gap because it offers the least opposition.

(*b*) One containing an electromagnet that will open the circuit very quickly if an abnormally large current flows through it. See Art. **170.**

(53) (*a*) In parallel.
(*b*) In parallel.

(54) The line resistance is $12.1 \times 7.93 = 95.95$ ohms. The resistance of four 30-ohm pony relays is 120 ohms,

and since it is required to make $r = l + B$, then $B = 120 - 95.95 = 24.05$; or, say, 24 ohms. By formula **11**, in which $\frac{s\,b}{p} = B$ and $e = 1$, we get

$$s = .1 \times (24.05 + 120 + 95.95) = 24 \text{ cells.}$$

One row of 24 cells will have 72 ohms internal resistance. Hence enough cells must be used to bring this resistance down to about 24 ohms. Since $24 = \frac{72}{3}$, then three rows, 24 in each row, making 72 cells in all, will be required. This could be done in another way. By solving formula **21** for N, and substituting in it the values given in the problem, we get

$$N = \frac{(2 \times 120 \times .1)^2 \times 3}{1^2 \times (120 - 95.95)} = 72 \text{ (nearly).}$$

By formula **18**,

$$s = \frac{2 \times .1 \times 120}{1} = 24 \text{ cells in series.}$$

By formula **20**,

$$p = \frac{72 \times 1}{2 \times .1 \times 120} = 3 \text{ rows in parallel.}$$

(55) (*a*) Place plugs in holes *2* and *9* (or in *3* and *8*), and in *18* and *24* (or in *16* and *22*, or in *17* and *23*).

(*b*) Place plugs in holes *22* and *5* (or in *4* and *23*) and in holes *8*, *15*, and *13* (or in *9*, *14*, and *13*).

(56) There are no loose or unused flexible cords on or under the board; every loop and desk set can have a jack at every switchboard in the office, and, consequently, all are available for use at each board without transferring and consequent loss of time. Also, the main lines can be arranged so that they can be transferred from one board to another by means of a multiple set of jack circuits that can be provided to connect all the main-line boards with a central board. See Arts. **222** to **224**, inclusive.

(57) The excessive heat generated in a coil of fine German silver wire when an abnormally large current flows through it will melt an easily fusible wax in which the coil is imbedded and thus the softening of the wax allows a spring to pull the coil out of place, break the wire, and open the circuit. See Art. **179.**

(58) Put the double-wedge terminal of a desk set in the spring jack belonging to line *5* and directly below the vertical strap *5*, and insert an ordinary plug in the hole *d–5*.

TELEGRAPHY.

(PART 2.)

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(1) Dip a wire from each terminal into a vessel containing acidulated water. Bubbles will arise from each of the wires while the current is flowing, but at a much greater rate from the wire connected to the negative than from the one connected to the positive terminal of the generator.

(2) (*a*) The solution is composed of water and sulphuric acid mixed to a density of 1.2, as indicated by an ordinary hydrometer.

(*b*) The solution should be mixed in an earthenware vessel and the acid poured slowly into the water, the mixture being thoroughly stirred in the meanwhile.

(*c*) In order to prevent the throwing of the acid in all directions by the sudden formation of steam due to the intense heat generated.

(3) An alternating current may be defined as a current that is continually reversing its direction in the circuit; consequently, if represented by a curve, this curve will lie alternately on both sides of the axis.

(4) It merely reduces the strength of the alternating current according to Ohm's law. It causes neither a lag nor a lead of the current relatively to the impressed electromotive force.

(5) (*a*) It is the maximum number of signals, or sometimes words, per minute that can be sent and detected at the receiving end of a line or cable.

(*b*) It depends on the product of the total resistance and the total electrostatic capacity of the line or cable, or the KR, as this product is usually called, and also on the arrangement and kind of apparatus used at the sending and receiving offices.

(6) See Art. **109.**

(7) (*a*) The principal advantage of dynamos and storage batteries over primary batteries lies in the fact that they are so much more economical.

(*b*) They require less attention and less room.

(8) A motor-dynamo is a combination of motor and dynamo mounted upon one base, with their shafts rigidly coupled together, the armature windings being distinct upon each shaft and each armature having its own independent field.

(9) See Art. **112.**

(10) There are 3 sets of 5 machines each; one set supplies a positive electromotive force, one a negative electromotive force, while the third set is held in reserve to replace either of the other two in case of accident. The 5 machines in each set are connected in series, thus giving 5 different potentials for use on the line wires. The fields of all 5 dynamos are excited by current from the machine carrying the smallest load. In series with each line wire just before it is connected to the bus-bar is a non-inductive resistance coil or incandescent lamp of about 2 ohms per volt.

(11) The use of low-resistance relays increases the working efficiency of the line and keeps the circuit, especially during wet weather, in better working condition. See Arts. **65, 66,** and **68.**

(12) (*a*) Copper, because it does not corrode or rust away.

(*b*) See Arts. **83** and **84.**

(13) The total area of the positive plates in a cell is determined either by the amount of current that is to be steadily taken from the cell, that is, by the rate of discharge, or by the ampere-hour capacity required.

(14) (*a*) Distributed capacity can only be neutralized by a distributed self-induction. Professor Pupin proposes to accomplish this result by placing coils having a certain definitely calculated inductance in series in the line or cable at certain regular and definitely calculated distances apart.

(*b*) No; because up to January, 1900, there was no commercially successful method for making a distributed self-induction that would accomplish the desired result; but in that year (1900), Professor Pupin devised a method that promises to prove successful.

(15) (*a*) See Art. **85.**

(*b*) Those from trolley circuits that seek the telegraph line wire as a return path.

(16) By an adjustable rheostat, capable of carrying the entire current on the motor side without overheating, connected directly in series with the armature.

(17) (*a*) Self-induction tends to make a fluctuating or alternating current lag in phase behind the electromotive force that produces it. It also tends to reduce the strength or amplitude of the current.

(*b*) Capacity tends to make the current lead the electromotive force in phase. It also reduces the strength or amplitude of the current, but the greater the capacity, the less does it reduce the current strength.

(*c*) Yes, under certain conditions.

(*d*) Electrostatic capacity can be made to neutralize self-induction *perfectly* (for ordinary low frequencies, that is, for frequencies below about 150 periods per second) for only

one particular frequency at a time. If the frequency is changed, then a different amount of capacity must be introduced in order to neutralize perfectly a given self-induction. Professor Pupin's method will accomplish this result close enough for the higher frequencies and approximately for about all frequencies.

(18) By using two line wires, instead of using the earth as a return circuit.

(19) By an adjustable rheostat in the field circuit of the dynamo or in the field circuit of the motor. In the latter case, the adjustable rheostat regulates the dynamo voltage by altering the speed of the machine and is not as desirable a method as the one first mentioned.

(20) Make sure that the positive pole of the battery is connected to the positive pole of the charging generator.

(21) When, in formula **7**, $2\pi n L = \frac{1}{2\pi n Q}$ (see Art. **34**), the inductance neutralizes perfectly the electrostatic capacity. Substituting in the above expression the values given in the question, we get

$$2 \times 3.1416 \times 300 \times 8 = \frac{1}{2 \times 3.1416 \times 300 \times Q}.$$

Therefore, $Q = .00000003518$ farad, or .03518 microfarad.

(22) (*a*) The amplitude of an alternating-current curve is the maximum height that the curve rises above or falls below the axis.

(*b*) The frequency of an alternating current is the number of complete cycles occurring in one second.

(*c*) The mistake is often made of referring to the frequency as the number of alternations or half-cycles that occur in a second, thus giving twice the proper value to the frequency.

(23) (*a*) The insulation resistance of a line is the joint resistance of all the leaks from that line to the earth and other conducting bodies.

(*b*) It depends on the resistance of the insulators and poles, the number of insulators and poles per mile, the length of the line, and the state of the weather.

(**24**) By connecting a resistance in series with the armature and the source of current, the resistance being gradually cut out as the armature comes up to full speed.

(**25**) See Art. **190.**

(**26**) (*a*) The impedance is the total opposition or the apparent resistance offered to the flow of an alternating current by a circuit possessing self-induction or electrostatic capacity, or both, in addition to the so-called simple ohmic resistance.

(*b*) See Art. **29.**

(*c*) See Art. **31.**

(*d*) See Art. **33.**

(**27**) (*a*) 50 milliamperes.
(*b*) 50 milliamperes.
(*c*) 100 milliamperes.
(*d*) 250 milliamperes.

(**28**) Local capacity is the capacity that is connected in a circuit at one definite point, while distributed capacity is, as its name indicates, distributed throughout the length of a line or cable. Distributed capacity is usually caused by the condenser action, that is, by the electrostatic induction, between line wires themselves or between one line wire and the ground or other conductors. Local capacity is usually formed by the insertion of an ordinary condenser at some point in the circuit.

(**29**) (*a*) See Art. **42.**
(*b*) About 24,000. See Art. **42.**

(**30**) To prevent the storage battery from discharging back into the charging circuit in case the electromotive force of the charging circuit falls below that of the batteries.

(31) It is a starting rheostat so arranged that if the current supply is shut off or the motor stops for any reason, the movable arm will be immediately returned to the "off" or starting position, so that all the resistance in the box will be in the circuit in case the current is turned on again before an attendant has time to turn the handle to the off-position; and, also, so that the attendant through carelessness or ignorance cannot start up the motor by closing a switch when the handle is in the on-position. It protects the motor from injury due to the large current that would flow if the arm was on the on-position before the motor could get up to full speed.

(32) (*a*) It makes the time-constant $\frac{L}{R}$ of the sounder circuit smaller than it would be were the sounder connected directly across the circuit without this resistance, and hence it makes the sounder act more promptly. Where it is desirable to use a high electromotive force (such as 110 volts), and sufficiently high resistance sounders are not used, the non-inductive resistance is used to keep the current from exceeding the proper value in any one line.

(*b*) It is not so economical, because more power is required. There is also the cost of the coils or lamps to be considered.

(33) It is a device for opening the circuit in case the current exceeds a certain maximum value or falls below a certain minimum value.

(34) (*a*) It is neither zero nor constant. It is a very variable quantity depending on the character of the soil and region, and on the area and material of the ground plates.

(*b*) It may usually be considered negligible when the total resistance of the one line wire and all relays is greater than about 1,000 ohms.

(35) Except in rocky or dry sandy regions, the resistance is usually concentrated entirely at the contact surface

between the ground plates and the soil in which they are buried.

(36) The line must be composed of two wires, the earth not being used at all. In cables, the two wires forming the two sides of one circuit must be spirally twisted around each other; on overhead circuits, the two wires must be occasionally transposed. See Arts. **91** and **93.**

(37) The time, in terms of the constant a, that it takes the current to become strong enough to be detected at the receiving end, is $\frac{1}{.2} = 5$. From Table 2, or the curve in Fig. 11, we obtain the value corresponding to $5a$ to be about 38.75 per cent. Then the actual strength of the current at the receiving end 1 second after closing the key at the other end is $.3875 \times 2 = .775$ milliampere.

(38) Because the charges can more easily leak to the earth over the surface of the insulators in wet weather than in dry; and, moreover, in dry weather more of the charge must get to earth through the terminal offices where the inductance of the relays oppose and hence delay the discharging of the line.

(39) (*a*) By the working efficiency of the line is meant the variation in the strength of the current at any station when the key at another station is alternately opened and closed.

(*b*) It depends on the ratio between the conductor resistance, which includes the line and all relays, and the insulation resistance.

(40) (*a*) The ratio per mile $= n^2\left(\frac{r}{s}\right)$. See Art. **60.**

$$n^2\left(\frac{r}{s}\right) = (30)^2\left(\frac{\frac{6}{30}}{25,000,000}\right) = \frac{1}{138,900}. \quad \text{Ans.}$$

(*b*) $$\frac{(500)^2 \times 30 \times 6}{25,000,000} = \frac{18}{10}, \text{ or } 1.8. \quad \text{Ans.}$$

(41) By adjusting the strength of the magnetic field. This is usually accomplished by means of an adjustable rheostat in the field circuit.

(42) See Art. **106.**

(43) The voltage of a given dynamo depends on the speed of rotation of the armature and the strength of the magnetic field.

(44) See Arts. **40** and **41.**

(45) The best way is to charge the storage batteries from a dynamo or from a motor-dynamo that will give the proper amount of current at the proper voltage. The motor dynamo may be supplied with current from a lighting or power circuit. Another way is to charge the batteries from an ordinary incandescent-light direct-current circuit, regulating the current by a proper rheostat, which may consist of a number of incandescent lamps. A small cell at a branch office may even be charged by current from the regular telegraph-line circuits leading to a large main office. See Arts. **191** and **193.**

(46) Defective insulation, crosses, breaks, and defective ground connections at one or both terminals.

(47) See Arts. **168** and **170.**

(48) To prevent injuring the storage battery or dynamo if an unusually large current should start to flow in the circuit on account of a short circuit somewhere on the system. The abnormally large current will melt the fuses or operate the circuit-breakers, opening the circuit in either case and so protect the apparatus.

(49) (*a*) Because the operation of one circuit is very apt to cause variations in the current in the other circuits and so cause more or less trouble.

(*b*) The disks have holes for the insertion of pegs only on one side, so that more than one vertical strap cannot be

connected to the same disk and to the same non-inductive resistance.

(50) A converter is a dynamo-electric machine having one armature, one field frame, and two commutators, or one commutator and one pair of collector rings. The armature conductors and commutator bars or collector rings are so connected together that a current going in through the commutator or one pair of collector rings at a certain potential is converted into a current of another strength and another potential as it comes out at the second commutator or pair of collector rings.

(51) (*a*) See Art. **99.**
(*b*) See Art. **100.**

(52) A shunt dynamo is a self-excited machine, the field coils of which (including the field rheostat) are connected across the terminals or brushes and are in shunt or parallel with the external circuit.

(53) In the expression for w in formula **9,** $r = \frac{5}{25} = \frac{1}{5}$, $s = 20{,}000{,}000$, $n\,l = 25 \times 400 = 10{,}000$.

Hence,

$$W = 2.718^{10{,}000\sqrt{\frac{1}{5} \times \frac{1}{20{,}000{,}000}}} = 2.718.$$

By formula **9,**

$$P = \frac{2}{2.718 + \frac{1}{2.718}} = .648.$$

Hence, 64.8 per cent. of the total current reaches the distant end.

TELEGRAPHY.

(PART 3.)

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(1) One of the most important considerations is the question of securing proper right of way. Directness of the route as well as the character of the soil, configuration of the country, and freedom from obstructions, such as trees, houses, and many other things. See Arts. **1** and **2.**

(2) To guy stubs placed in the ground at a short distance from the pole, to anchor logs buried in the ground, or to trees or rocks when available. See Arts. **48, 49, 50,** and **51.**

(3) (*a*) $\frac{1}{1000}$ inch.

(*b*) A circular mil is a unit of area used in expressing the cross-section of round or cylindrical wires. It is equal to the square of the diameter in mils.

(*c*) Because the area of a round wire in circular mils is equal to the square of the diameter in mils, and because the conductivity of wires of the same material are directly proportional to their area expressed in circular mils. The odd number $\pi = 3.1416$ does not have to be considered where circular mils can be used.

(4) (*a*) Norway pine, chestnut, white cedar, and cypress.

(*b*) See Table 1, Art. **3.**

(*c*) White cedar and chestnut.

(5) The selection of the positions for the poles on the river banks should be made with due regard to obtaining a firm foundation. After setting the poles, they should be thoroughly guyed in a direction to meet the strain of the long span between them.

(6) $80.808 \times 80.808 = 6,529.95$ circular mils. Ans.

(7) That poles 30 feet and over in length shall have tops at least 22 inches in circumference. Sometimes, however, the circumference 6 feet from the butt is also specified, the standard dimensions being given in Table 2.

(8) (*a*) So proportioning the heights of the various poles and their location with regard to depressions or elevations in the ground that no sharp vertical bends will occur in the line wires when stretched.

(*b*) Some poles will be subjected to very severe downward strains, while others will be subjected by the line wires to upward strains that tend to lift them out of the ground or to pull off their insulators.

(9) (*a*) Cross-arm braces are flat strips of iron $\frac{1}{4}$ in. by $1\frac{1}{4}$ in. in cross-section, and from 20 to 30 in. long.

(*b*) They are used to prevent the cross-arm from twisting in the gain, thus keeping them in a horizontal position.

(*c*) They are usually attached to the pole by a single lag-screw passing through the two braces of one arm. They are attached to the cross-arms by 4-inch carriage bolts that pass entirely through the cross-arm and are secured by nuts. Washers should be placed under both the heads and the nuts of all bolts.

(10) (*a*) The pole is braced by four pike poles. The pikes should be struck in the pole at points about 8 feet from the ground, while the butt ends of the pike poles are planted in the ground.

(*b*) The filling in should not be done faster than the earth can be properly tamped.

(11) $\frac{1}{8}$ in. $= 125$ mils. $125 \times 125 = 15{,}625$ circular mils. Ans.

(12) (*a*) The butts should be pointed down hill.

(*b*) So that the tops of the poles will have a shorter distance to travel while the pole is being raised, making the work of raising easier.

(13) (*a*) Locust is the best, although oak is much used.

(*b*) The shank of the pin is driven into the hole in the cross-arm and secured by a nail driven through the cross-arm and the pin.

(*c*) Iron and steel pins.

(14) When the ground is marshy, when for any reason the hole cannot be dug to the required depth, or where the pole is to be subjected to some very severe strain, as in dead-ending. See Arts. **42, 43,** and **55.**

(15) See Art. **64.**

(16) $\sqrt{10{,}816} = 104$ mils. Ans.

(17) (*a*) Sometimes the poles are impregnated with creosote or with coal tar and carbolic acid after the butts have been charred. In this country the poles are frequently coated for a distance of 6 or 7 feet from the butt with hot pitch. See Arts. **7** and **8.**

(*b*) The wind-and-water line is the line around the pole at the surface of the ground, so called because it is on this line that the combined effects of the air and moisture are greatest.

(18) They should be given two or three coats of best white lead, in order to prevent moisture from entering the grain of the wood.

(19) (*a*) See table in Art. **32.**

(*b*) The nature of the soil, the height of the poles, the number of wires to be carried, the number of poles to the

mile, the frequency and intensity of heavy wind or sleet storms, and the side or end strains placed on the poles by the wires in turning corners, or in dead-ending.

(20) (*a*) A guy that branches at a point near the pole, one branch extending to the top of the pole and the other to a point at or near the lowest cross-arm.

(*b*) In order to prevent any undue strain occurring in the vicinity of the lower cross-arm. See Art. **44.**

(21) (*a*) By different gauges, referring to the various sizes by number; usually, the larger the wire, the smaller is the gauge number.

(*b*) The Brown & Sharpe, or American, gauge, usually abbreviated to B. & S.

(*c*) The Birmingham wire gauge, usually abbreviated to B. W. G.

(22) The cross-sectional area of the wire is doubled as the gauge number is diminished by 3, and is halved as the gauge number is increased by 3. See Arts. **74, 75,** and **76.**

(23) See Art. **77.**

(24) (*a*) "Extra Best Best," "Best Best," "Best," and "Steel."

(*b*) See Art. **96.**

(25) Because if it is drawn too tight in warm weather, there will be no room for contraction in its length during the cold weather, and it is therefore likely to break by its own efforts to contract.

(26) (*a*) Saturated- and dry-core cables.

(*b*) Dry-core cables have a very low electrostatic capacity per mile and a very high insulation resistance. They are, however, very susceptible to injury by moisture, and should only be used where very low electrostatic capacity is an

absolute necessity, as in telephone cables. The saturated-core cables have a higher electrostatic capacity per mile than the dry-core cables, but have the advantage of not being so susceptible to moisture, and are, therefore, more desirable for telegraph purposes.

(27) They are supported from messenger or carrier wires either by wrapping the cable and messenger wire together with marline twine, or by means of hooks fastened around the cable and hooked to the messenger wire. See Arts. **177, 184,** and **193.**

(28) (*a*) It is the weight of a wire of uniform size 1 mile long and having a resistance of 1 ohm.

(*b*) See Art. **84.**

(29) (*a*) At least 3 times its weight per mile. Some companies require the breaking strength of hard-drawn copper wire to be 3.2 times its weight per mile. See Art. **91.**

(*b*) See Art. **92.**

(30) (*a*) It was not properly galvanized and should be rejected.

(*b*) Because the zinc coating was removed by the action of the solution, allowing the copper in the solution to deposit directly upon the iron. See Art. **100.**

(31) (*a*) See Arts. **60, 126, 129, 130,** and **131.**

(*b*) The solder should be applied at the center of the joint only, for the heat that is necessary to cause the solder to flow is likely to weaken the line wire, and it is better to weaken it where it is double than at the ends of the splice where it is single.

(32) (*a*) It is proof against all chemical action and will last indefinitely unless destroyed by mechanical means. It is a good insulator, is cheap, and is easily laid.

(*b*) Generally the single-duct section; there are some, however, who prefer the multiple-duct.

(*c*) It is generally considered easier to manufacture, is more readily handled, and is more flexible when turns have to be made.

(33) A pole is located near the manhole and a wrought-iron pipe is led from the manhole up the side of the pole, the cable passing through this pipe into a pole box and terminating in a cable head. The cable wires are connected through the cable-head terminals, lightning arresters, and spider wires to the overhead circuits.

(34) $\frac{869 \times 100}{885} = 98.19$ per cent. Ans.

(35) See Arts. **106, 107, 108, 111,** and **113.**

(36) (*a*) The wires are bunched into cables. See Art. **137.**

(*b*) Cables in which each copper conductor is insulated with saturated paper or cotton fiber, the bunch of insulated conductors is then wrapped with insulating material and the whole covered with a lead sheath.

(37) In order to allow sufficient length for making necessary splices in the future.

(38) By means of a mandrel drawn through the conduit during the process of laying; also, in the multiple-duct clay conduit, by dowel-pins at the joint between each two sections.

(39) (*a*) At the points where the earth currents flow from the cable sheath to the earth or other conductors.

(*b*) See Arts. **224** and **225.**

(40) Using formula **2,** in which $W = \frac{d^2}{62.5}$, we have

$$\text{weight} = \frac{71.96^2}{62.5} = 82.85 \text{ lb.} \quad \text{Ans.}$$

(41) Copper wire possesses at least six times as great a conductivity as iron and is non-magnetic; therefore, copper

line circuits possess less impedance than iron line circuits. Copper wire is far more durable than galvanized iron or steel wire. See Art. **104.**

(42) (*a*) The coil of wire is placed upon a reel on a hand-barrow, and one end of it fastened at the beginning of the section to be strung. The hand-barrow is then carried along, the wire being paid out from the reel as it proceeds. The wire is then lifted up to the cross-arms on each pole, and afterwards is pulled up and tied. Another method is to place the reel at the starting point and pull the wire along over the cross-arms of the section to be strung.

(*b*) See Art. **117.**

(43) For connecting the wires of a cable to other wires without allowing moisture to enter the insulation at the cable end.

(44) (*a*) The creosoted-wood or pump-log conduit.

(*b*) The fact that it will probably decay eventually, and also because there seems to be an acid formed that sometimes attacks the lead sheath of the cable.

(45) See Art. **209.**

(46) The danger point or points should be located by means of tests with a voltmeter, as explained in Arts. **224** and **225,** and at these points the cable sheath should be bonded, that is, connected, to the neighboring conductors to which the current is flowing. See Arts. **226, 227,** and **228.**

(47) Using formula **3,** Art. **87,** we have the resistance per mile $= \frac{56,970}{101^2} = 5.58$ ohms. Ans.

(48) (*a*) Galvanizing is the process of coating wire with a thin film of zinc. It is used only in connection with iron wire, the wire being first cleaned by being passed through hydrochloric acid, after which it is drawn, while hot, through a vat of molten zinc. See Art. **99.**

(*b*) It is not necessary to galvanize copper wire because an oxide of copper forms on the surface of the wire when it is first exposed to the weather. This oxide is insoluble in water and is not attacked by any gases usually found in the air. When this very thin film is once formed, there is no further decomposition or wasting away of the copper wire.

(49) The tension may be measured by a line dynamometer attached to one end of the wire, by which the pull in pounds may be directly read, as on a spring balance. Another and more common method is to judge of the proper tension by the amount of sag in the spans, by sighting from cross-arm to cross-arm, or otherwise. The proper amount of sag for spans ranging from 75 to 200 feet in length is given in Table 22.

(50) (*a*) First, the wires may be placed farther apart; second, the wires may be made smaller; third, an insulating medium having a lower specific inductive capacity may be used.

(*b*) The third. The first method makes the cable too bulky; the second may be used to some extent, but if carried to an extreme, the resistance of the wires is made too high and the tensile strength too small. The method involving the use of an insulating material having a low specific inductive capacity affects neither the strength nor the size of the cable appreciably, and is, therefore, the most desirable.

(51) (*a*) See Art. **203.**

(*b*) By beveled socket joints of cast iron on the ends of the sections.

(52) (*a*) To allow access to the cables at frequent intervals in order to facilitate the drawing in or out of the cable and the splicing and repairing.

(*b*) About 400 feet apart.

(*c*) At the corners of streets and at short turns in the conduit.

(53) (*a*) 25 feet.

(*b*) 12 feet.

(54) Double cross-arms should be put on all office poles, on poles at railway and river crossings, at corners, on unusually long sections, on heavy lines on the first pole back from a corner.

(55) (*a*) No. 8 B. W. G. iron wire.

(*b*) At least every fifth pole.

(56) Because the grain of the pole is not injured as much by the carriage bolts for which holes are bored as by the lagscrews, which tend to tear the fiber of the wood; furthermore, the replacing of old cross-arms by new ones is rendered easier.

(57) Sometimes of two No. 9 iron wires twisted together, sometimes of a single No. 8 or No. 6 iron wire, and sometimes of a steel rope composed of 7 strands and varying in external diameter from $\frac{3}{16}$ to $\frac{1}{2}$ inch.

(58) (*a*) 18 feet.

(*b*) Tubular hard rubber, rectangular iron box, and pothead terminals.

(59) (*a*) See Art. **163.**

(*b*) The rising of bubbles through the insulating liquid is a good sign of moisture. See Art. **154.**

(60) (*a*) About 6.

(*b*) Long-handled digging spoon, long-handled round-pointed shovel, a combined crow and digging bar, and in fair earth or wet places a post-hole auger.

TELEGRAPHY.

(PART 4.)

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(1) A telegraph repeater may be defined as an arrangement of apparatus for repeating signals from one main line into another main line. It is virtually a relay that is controlled by the sending operator at the end of one main line, and which, in turn, controls the second main line and hence the relay at the far end of it; the repeater itself is located at about the middle point of the distance covered. See Art. **1.**

(2) (*a*) The arm of the switch must be turned to the left, so as to connect *c* and *d*.

(*b*) Suppose that all circuits are closed and that the switch *k* is turned to the left, and that the eastern operator, in order to start sending, opens his key. This allows relay *R* to open, thus allowing sounder *S* to open. This opens the west line at *f*, and thus, since it is also open at *b*, the relay R_1, and also the relay at the distant western station, will open. The east line cannot open at the repeater because it is closed between *c* and *d* by the switch arm *k*. When the eastern station closes his key, the relay *R* and sounder *S* will close, thus closing the west line at *f*.

(3) A button repeater is one requiring that a button or switch should be turned by the hand of an operator at the

repeater in order to change from repeating in one direction to repeating in the other direction.

(4) With the switch *g* closed and *k* connecting *a* with *d*, the west line may repeat into the east line; with *g* closed and *k* connecting *b* with *c*, the east line may repeat into the west line; with *g* closed and *k* connecting *c* and *d*, the east line and the west line may be used independently; with *g* open and *k* connecting *c* and *d*, the west line and the east line are connected straight across.

(5) The sending should be heavy or firm; that is, the signals should be somewhat prolonged, because the current requires time to rise from zero to its maximum and to fall again to zero on account of the electrostatic capacity of the line and the inductance of the relays, and because time is also required for the various armatures to move across the gap between the two stops.

(6) Line from San Francisco to New Orleans, a distance of 2,484 miles, with one repeating station. See also Art. **4.**

(7) There are three reasons: *First*, as a line increases in length, the working efficiency decreases until it becomes so small that satisfactory signals cannot be transmitted no matter how much the battery power is increased. *Second*, as a line increases in length, the resistance increases, and, consequently, the electromotive force must be correspondingly increased, assuming that the insulation remains perfect; if it does not, the electromotive force must increase faster than the resistance. But it is impractical to use over 400 volts as an extreme limit, and 300 is usually considered very high for single working. *Third*, as a line increases in length, the electrostatic capacity increases until the latter seriously diminishes the speed of signaling. These three causes combine to limit the length of line over which it is practical to signal without using repeaters. See Art. **2.**

(8) Button repeaters are generally used only for temporary purposes.

(9) (*a*) An automatic repeater is one that will *automatically repeat in either direction*, that is, it does not require an operator at the repeater to turn a switch when the direction of sending is to be reversed.

(*b*) To adjust the instruments and care for the batteries. Besides doing other work, one operator may look after a number of repeater sets.

(10) (*a*) An artificial line is a branch circuit to the ground having the same resistance and capacity as the line. The resistance and capacity must be properly arranged so that the artificial line will not only have the same resistance and capacity but will also charge and discharge at the same rate as the line.

(*b*) It is used in order to make the current from the home battery divide equally through the line and artificial line, so that it will not energize the differentially wound relay or relays at the home station.

(11) The chief function of an automatic repeater is to keep the sending circuit at the repeating station closed as long as this circuit is repeating into the other or receiving circuit. See Art. **15.**

(12) (*a*) See Art. **71.**

(*b*) See Art. **72.**

(13) (*a*) Closed.

(*b*) When the key at the western station is opened, the western relay of the repeater Fig. 3 opens the local circuit through the magnet S_1 of the transmitter T_1, because M_1 does not release its armature. The transmitter T_1 breaks two contacts, one slightly before the other. The contact x_1 that is broken first opens a local circuit through the extra magnet M on the opposite, or eastern, side, causing this extra magnet to release its armature and hold the contact at y closed. Thus the transmitter T on the same, or eastern, side and the western line is held closed. The second

contact a_1 that is broken at the western transmitter T_1 opens the eastern main line, but the armature g of the eastern relay R is not released, although the eastern relay R is demagnetized, and, hence, the local circuit controlled by the armature g is not opened, because the armature is held against the contact or front stop y by the stronger spring s that acts on the armature o of the extra magnet. Hence, the armature of the eastern relay R always remains against its front stop y and, therefore, keeps the transmitter T on the same, or eastern, side, and, consequently, the western line, closed at a while the western key is being operated. Moreover, the circuit through the extra magnet M_1 is kept closed at x by the eastern transmitter, thus allowing the western relay R_1 to have full control of its armature g_1.

(14) (*a*) On the principle of the Wheatstone bridge.

(*b*) The battery, key, and artificial line.

(*c*) The galvanometer.

(15) Only enough movement to break the circuit, or about $\frac{1}{32}$ inch. See Art. **21.**

(16) It was used to obviate or reduce the mutilation of signals made on the neutral relay when it should remain closed. This breaking up of a signal is due to the interval of no magnetism in the neutral relay when the distant pole changer reverses the distant full (or long end) main battery. See Art. **165.**

(17) In a double contact between the tongue of the transmitter and both the upper and lower contact points, due to an improper adjustment of the transmitter. See Art. **267.** If the system was previously used as a duplex, the small margin may be due to an extra resistance that was then inserted in the battery circuit, but it should have been cut out before attempting to again work the system as a quadruplex. See Art. **268.**

(18) See Art. **204.**

(19) (*a*) A side-line repeater is one arranged to repeat from a main line that runs through a repeating station into a side line that branches off from the repeating station.

(*b*) Most any single-line automatic repeater given, except, perhaps, the Toye, may be used as a side-line repeater.

(20) This condition exists for four different combinations of the four keys, which are as follows : All four keys open; all four keys closed; the pole-changer keys Pk and Pk_1 closed and the transmitter keys Tk and Tk_1 open at both terminal stations; the pole-changer keys Pk and Pk_1 open and the transmitter keys Tk and Tk_1 closed at both stations. See Fig. 69 and combinations 1, 6, 11, and 16 in Table 3.

(21) The Toye repeater depends for its operation on the substitution of a resistance equal to that of the receiving line in the place of the latter at the instant that the receiving circuit is opened, the resistance being substituted in place of the receiving line in such a manner as to hold closed the relay and transmitter that control the sending circuit, and, consequently, holding the sending circuit itself closed at the repeater. See Art. **23.**

(22) (*a*) In the normal condition of the Neilson repeater, which is shown in Fig. 10, the relays R and R_1 and the transmitters T and T_1 are closed and the repeating sounders RS and RS_1 are open.

(*b*) The relays R_1 and R, the transmitter T_1, and the repeating sounder RS are open. The repeating sounder RS_1 and the transmitter T are closed.

(23) The Toye repeater is extremely simple, and requires comparatively few pieces of only standard apparatus. On the other hand, it is hard on the batteries, hard to keep adjusted in changeable weather, and is not readily available as a side-line repeater. See Art. **25.**

(24)

TABLE 3.

NINTH COMBINATION.

No.	Keys.				Pressure at Point.		Current in			Effective Current.		Relays Operated.			
	West.		East.		h. West.	h_1 East.	West.	Line.	East	West.	East.	West		East.	
	Pk.	Tk.	Pk_1.	Tk_1.			$A L$.	L.	$A L_1$.	Relays $N R$ and $P R$.	Relays $N R_1$ and $P R_1$.	$P R$.	$N R$.	$P R_1$.	$N R_1$.
9	Open	Open	Open	Closed	−100	−300	$\frac{100}{R}(x)$	$\frac{200}{R}(y\ x_1)$	$\frac{300}{R}(x_1)$	$\frac{300}{R}(y\ L)$	$\frac{100}{R}(x_1\ A\ L_1)$	Open	Closed	Open	Open
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

The following explanation will show how to determine the direction of the effective currents in the line and artificial-line coils of the relays used in the quadruplex system. It is necessary to bear in mind that PC operated by the key Pk is the pole changer, and serves to control only the *polarity* of the current that is directed toward the line. T, operated by the key Tk, is the transmitter, and controls only the *strength* of the current, *irrespective of its polarity*. Thus, when the lever of the pole changer PC is unattracted, the polarity of the current directed toward the line will be negative no matter what position the lever of the transmitter T occupies. When the lever of the pole changer PC is attracted, the polarity of the current sent to the line will be positive no matter what position the lever of the transmitter T occupies. In a similar manner, the voltage impressed upon the line will be the smaller of the two if the lever of the transmitter is unattracted, and the larger if it is attracted, regardless of the position of the lever of the pole changer PC. In working out the table, reference should be made to Fig. 54 or Fig. 55, and it should be remembered that the potential at the points h and h_1 may always be considered as determined by the positions of the keys at the corresponding end of the line. Take, for instance, the 9th set of combinations, the one left unsolved in Table 3. Both of the western keys are up, that is, open. The fact that the key controlling the pole changer PC is open determines the fact that the polarity is negative, and the fact that the key controlling the transmitter T is open determines the fact that its voltage is 100 and not 300. We therefore have -100 as the potential at h. In a similar manner, the fact that the key controlling the pole changer PC_1 at the eastern end of the line is up determines the fact that the polarity is negative, while the fact that the key controlling the transmitter T_1 is closed determines the fact that the voltage is 300 and not 100; thus, we have a potential of -300 at the point h_1 at the eastern end of the line. The determination of the direction in which the current flows in either branch is now a simple matter. Inasmuch as

the potential at the point h at the western end of the line is -100, we know that current will proceed from the battery B_1, Fig. 54, to earth at G, then to the earth at G_1, and up through the artificial line to the point h and through f, m, d, b, v, z, and q, back to the battery B_1. This direction of the current through the artificial-line circuit corresponds to the arrow x. The strength of the current in amperes will be, according to Ohm's law, 100 volts divided by R ohms, that is, $\frac{100}{R}$, R being the resistance of the artificial-line circuit. We therefore obtain for the current in the western artificial line $A\,L$ the value $\frac{100}{R}\,(x)$ for insertion in column 8. To determine the line current, we have only to calculate the difference of potential between the points h and h_1 at the eastern and the western end of the line. Evidently, the difference of potential is 200 volts, and, therefore, current will flow from west to east in the line, its direction corresponding to the arrows y and x_1 and its strength by Ohm's law is 200 divided by R. This current does not flow through the artificial line at either end, but through the batteries, pole changers, and transmitters at each end and through the line itself. We have, therefore, in column 9 for the line current, $\frac{200}{R}\,(y\,x_1)$. The potential at the point h_1 at the eastern end of the line is -300. Therefore, current will flow from the whole battery B_2 and B_3 in series to ground G_2, and up through the artificial line $A\,L_1$ and back in the direction of the arrow x_1 to the battery. The strength of this current will be $\frac{300}{R}$. We therefore have for the current in the eastern artificial-line circuit $\frac{300}{R}\,(x_1)$ for insertion in column 10. Examining now the conditions with a view to determining the effective current in the set of relays, we find that at the western end there is a current of $\frac{200}{R}$ in the direction of the arrow y in the line, while there is

a current $\frac{100}{R}$ in the artificial line $A\,L$ in the direction of the arrow x. These two currents are of such direction as to add their effects; hence the effective current is equivalent to $\frac{300}{R}$ (y) in the line coils and zero current in the artificial-line coils of the relays. We therefore have for the effective current in the western relays $\frac{300}{R}$ $(y\,L)$ for insertion in column 11. At the western station this current, being strong enough, closes the neutral relay $N\,R$, but it is not in the proper direction to close the polar relay $P\,R$. At the eastern end of the line we have a current of $\frac{300}{R}$ in the artificial line in the direction of the arrow x_1 and of $\frac{200}{R}$ in the line in the direction of the arrow x_1. These currents, both flowing toward the point h_1, act differentially on the relay coils, and, hence, their difference must be taken. There is, therefore, a predominating current of $\frac{100}{R}$ in the artificial line $A\,L_1$ in the direction of the arrow x_1. The effective current in the relays at the eastern end of the line and to be inserted in column 12 is, therefore, $\frac{100}{R}$ $(x_1\;A\,L_1)$. This current is not strong enough to close the neutral relay $N\,R_1$ nor in the proper direction (see Art. **153**) to close the polar relay $P\,R_1$ at the eastern station.

(25) (*a*) The distinctive feature of the Weiny-Phillips repeater is the construction and winding of the extra magnets. They are differentially wound, and consist of one core with its winding enclosed in a soft-iron cylinder in such a manner as to form almost a closed path for the lines of force and, hence, make a very efficient electromagnet. The core is not magnetized when equal currents flow in both windings.

(*b*) Closed.

(26) (*a*) A single-current system is one like the Morse, in which the current flowing in either direction will make dots and dashes, spaces being made by breaking or stopping the current.

(*b*) A double-current system is one in which a current in one direction produces dots and dashes, a current in the opposite direction being required to terminate a dot or dash and start a space.

(27) When the eastern key in the Weiny-Phillips repeater is open, there is no current flowing through the relay magnet R, but there is current flowing through both coils of M; hence, the circuit is open at y. Consequently, the transmitter T is open. This opens at x one coil of M and at a the western line that passes through R_1. Consequently, M_1 is energized and although R_1 is deenergized, nevertheless the circuit through the magnet of the western transmitter T_1 is held closed by the armature at the front stop of the relay R_1. Therefore, T, M, and R are not energized in such a manner as to hold their armatures closed, but the western transmitter T_1 and the extra magnet M_1 are energized and hold their armatures closed in spite of the fact that there is no current through R_1.

(28) The distinguishing feature of the Horton repeater is the holding of the circuit of the transmitter on the sending side closed by the force of gravity, which alone acts on the relay armature when neither the relay nor the extra magnet are energized.

(29) Polarized relays are necessary in double-current systems.

(30) (*a*) In the normal condition, all instruments in the Atkinson repeater are closed.

(*b*) All circuits, except the two ending at f and f_1, are closed.

(*c*) The circuits through the magnets of the relay R, the transmitter T, repeating sounder RS_1, and the relay R_1

are open, and the circuits through the magnets of the transmitter T_1 and repeating sounder $R\ S$ are closed. The western circuit is open at a and local circuits are open at m, f, d, and m_1. The eastern circuit, which is open at the distant eastern station, is closed at a_1, and local circuits are closed at f_1 and d_1. See Fig. 16 and Art. **44.**

(31) On all submarine cables, on polar duplex, quadruplex, and Wheatstone automatic systems, and more or less on simplex land circuits throughout Europe.

(32) A polarized relay is one that requires the direction of the current flowing through it to be reversed in order to move the armature from one stop to another. See Art. **52.**

(33) In the Horton repeater, shown in Fig. 17, when the eastern key is open, current is flowing only through the magnets M_1 and T. Circuits are open at the repeater at f_1, d_1, and a_1 and closed at f, d, and a.

(34) When the capacity of the artificial line has been so adjusted that it is equal to the capacity of the line and, furthermore, when it charges and discharges at exactly the same rate as the line, a static balance is said to have been obtained. See Art. **120.**

(35) See Art. **136.**

(36) (*a*) Resistances, called retarding coils, may be connected in series with the condenser, one terminal of the condenser being connected to the ground and one terminal of the retarding coil to the line side of the rheostat. Another way is to connect the condenser to some point in the artificial-line rheostat instead of connecting it through a separate retarding coil. See Arts. **88** and **89.**

(*b*) The static balance in the first arrangement is obtained by adjusting the capacity of the condenser and the amount of resistance in the retarding coils; in the second arrangement, by adjusting the capacity of the condenser and the position of the point in the rheostat to which the condenser is joined. See Arts. **88** and **89.**

(37) (*a*) and (*b*) See Art. **95.**

(38) See Art. **96.**

(39) (*a*) The polar duplex is superior to the Stearns duplex because a polarized relay is more efficient and satisfactory than a neutral relay, especially in wet weather when leakage is troublesome. Moreover, on account of using currents that flow alternately in opposite directions, there should be less trouble due to the electrostatic capacity of the line wire. See Arts. **53** and **97.**

(*b*) The essential feature of the polar duplex is the differentially wound polarized relay. Since a polarized relay is used, a pole changer is consequently a necessity.

(40) (*a*) A continuity-preserving pole changer is preferable to one that opens the circuit in the act of reversing the direction of the current because in multiplex systems it is an advantage to preserve an uninterrupted path to the ground at the home station. This reduces what might otherwise cause an interruption in the signals coming from the distant station.

(*b*) Because the continuity-preserving pole changer would connect two dynamos in series through a comparatively small resistance and cause severe sparking at the contact points of the pole changer, when, as usual, machines of rather high voltage are required for duplex and quadruplex circuits. This severe sparking would very soon injure the contact points of the pole changer and put it in an unworkable condition. See Art. **112.**

(41) Where dynamos are used, the same machine supplies all circuits requiring the same polarity and about the same electromotive force; hence, where more than one line is supplied from the same machine, it is not practicable to reverse the direction of the current from one machine through one line without also reversing the current in the other lines. Batteries can readily be reversed, because a separate one is used in each line circuit, but a separate

dynamo must be used for each polarity, although one dynamo usually supplies current for a large number of lines. The positive pole of one machine and the negative pole of the other machine are permanently grounded, and the line is shifted from one pole of one machine to the opposite pole of the other machine when it is desirable to reverse the direction of the current in the circuit.

(42) (*a*) The ground coil is a resistance that is cut into the circuit to replace the transmitting apparatus and battery or dynamo when the system is being balanced.

(*b*) It is equal to the resistance of the circuit through the transmitting apparatus and the dynamo or battery to the ground. Where batteries are used, this resistance is practically equal to the internal resistance of the whole battery; and where dynamos are used, it is practically equal to the non-inductive resistance connected directly in series with the dynamo.

(*c*) It is used when the system is being balanced in order to keep the resistance of the circuit to the ground the same whether the transmitting apparatus and the battery or dynamo are cut in or out of the circuit.

(43) (*a*) The receiving circuit includes all apparatus and branch-office lines that may be connected through and controlled by the armature of the polar relay or the repeating sounder that is, in turn, controlled by the neutral relay. It is, consequently, a circuit including all the instruments that are controlled by the motions of the armature of either relay. Messages received on the relay are therefore repeated by all instruments connected in the receiving circuit.

(*b*) The sending circuit includes all instruments and branch-office lines that are connected in series with the magnets of the pole changer or transmitter; hence, the operation of a key anywhere in this circuit will operate the pole changer or transmitter, as the case may be, thus sending the message through the multiplex system to the distant office.

(**44**) *First*, to center the armature of the polar relay. *Second*, to obtain a resistance balance. *Third*, to obtain a static balance. *Fourth*, to adjust the pole changer.

(**45**) (*a*) To so adjust the position of the armature that it will remain against either stop or move with equal force from the middle position toward one side or the other. This is done so that the permanent magnetism will pull the armature with equal force from the center position toward either side.

(*b*) See Art. **120.**

(**46**) When the artificial line has been adjusted so that its resistance is exactly equal to that of the line, a resistance balance is said to have been obtained. See Art. **120.**

(**47**) By the neutral relay, because it is the instrument that is apt to give the most trouble. If a balance is based on the action of the polar relay, it may prove to be a false one when tested by the neutral relay; whereas, a balance based on the neutral relay will generally prove correct when tested by the polar relay.

(**48**) (*a*) The bridge duplex requires more battery power than the differential duplex to produce the same strength of current in the distant relay.

(*b*) The bridge duplex is superior to the differential duplex in that it requires less condenser capacity in the artificial lines, and the resistances and condensers can be more readily adjusted to suit the varying conditions of the line.

(**49**) All batteries or dynamos for supplying current to the main-line circuit are located at one end. No batteries or dynamos are required at the other end, except for the operation of the local receiving and sending circuits.

(**50**) A continuity-preserving pole changer is a device for reversing the direction of the current in a circuit without opening the circuit.

(51) The trouble would most likely be found in a defective or improperly adjusted transmitter, dirty contact points, a defective leak coil, or a loose connection of the wire attached to the leak coil at the distant station.

(52) By the diplex system. One message is sent by operating a pole changer that controls the direction of the current, and is received by a polar relay at the distant station. Another message is sent by operating a transmitter that controls the strength of the current. The transmitter is located at the same station as the pole changer. The message sent by means of the transmitter is received at the distant station by a neutral relay.

(53) (*a*) See Art. **162.**

(*b*) No. 1 is the polar and No. 2 the neutral, or common, side.

(54) It is advantageous in that it tends to force more of the incoming current through the artificial-line coils of the relays to ground, and it allows the strength of current to be made more nearly the same on short and long lines that are supplied with current at the same potential. On the other hand, a higher potential is required than would be necessary without it in order to give the same current in the short line circuit.

(55) (*a*) and (*b*) See Art. **162.**

(56) The resistance and total current supplied by the dynamos remain constant in all positions of the keys, but two resistances and the transmitter are so arranged that in one position of the transmitter, enough current is shunted to the ground so that the proportion of the total current that passes through the line is only $\frac{1}{3}$ or $\frac{1}{4}$, as the case may be, of what it would be in the other position of the transmitter. See Fig. 61 and Arts. **176** to **183,** inclusive.

(57) Four dynamos are used, two of one electromotive force, say 135 volts, and two of another, say 375 volts. The negative brush of one and the positive brush of the other

of the 135-volt dynamos, and the negative brush of one and the positive brush of the other of the 375-volt dynamos are permanently grounded. Then, by means of a special pole changer, two wires that run to the transmitter are connected in the closed position of the pole changer, as follows: One wire to the ungrounded positive brush of the 135-volt dynamo and the other wire to the ungrounded positive brush of the 375-volt dynamo. When the pole changer opens, the two wires are shifted to the ungrounded negative poles of the other two machines. The transmitter when closed connects one of the two wires mentioned to the line and when open it connects the other wire to the line. Thus, the pole changer controls the direction of the current flowing toward the transmitter, and the transmitter the strength of the current flowing toward the relays and line. See Art. **201.**

(58) Two machines of the same voltage but opposite polarity are used. Three resistance coils and one side of a double transmitter are so connected with the ungrounded terminal of the dynamo and to one wire leading to the pole changer as to cause only $\frac{1}{3}$ (or $\frac{1}{4}$ if the ratio is 1 to 4) as much current to flow toward the pole changer in the open as in the closed position of the transmitter. One side of the double transmitter controls the strength of current from the positive machine, the other side controls the strength of current from the negative machine. The pole changer merely shifts the line from one to the other of the two wires that connect through opposite ends of the double transmitter with the resistances and the dynamos of opposite polarity. See Fig. 75 and Art. **222.**

(59) Multiplex telegraphy is the transmission of two or more messages over the same wire at the same time.

(60) (*a*) As each circuit in a repeater is closed, a short delay occurs in the transmission of a message, for each armature moves over a short distance before the circuit is complete. This shortens the dots and dashes in proportion

to the number of contacts to be closed, and thus the dots are sometimes wholly lost. Therefore, in operating such a circuit, the dots and dashes should be made longer, or, as operators term it, the "sending should be heavy."

(*b*) By sending "heavy."

(61) (*a*) See Art. **73.**

(*b*) On changes both in the direction and in the strength of the current.

(*c*) One key at each end of the line governs the direction of the current, and the other key its strength. At each end there are two differentially wound relays, one of which (the neutral relay) is operated only by the changes in the strength of the current, while the other (the polar relay) is operated only by the changes in the direction of the current. The key at one end governing the strength of the current, therefore, produces at the other end no effect on the polar relay, because the latter responds only to changes in the direction of the current, but this key does operate the neutral relay because the latter responds to changes in the strength, but not to changes in the direction, of the current. Similarly, the key governing the direction of the current operates only the polar relay at the other end because the latter is affected by changes in the direction of the current. The home relays are not operated by the home keys because both relays are differentially wound.

(62) See Art. **251.**

(63) See Art. **259.**

(64) Messages are sent from the battery station to the distant station by the operation of a pole changer, a polarized relay being used at the distant station for receiving these messages. Messages are sent from the distant to the battery station by the operation of a transmitter that increases and decreases the resistance of the line circuit so as to decrease and increase the current in the ratio of 1 to 4.

These messages are received at the battery station on a specially connected double-wound neutral relay.

(65) See Art. **253.**

(66) The space between the armature and the left-hand magnet should ordinarily be at least twice as great as that on the right. The adjustment of the retractile spring is identical with that of an ordinary neutral relay. See Art. **191.**

(67) They are polarized instruments and no springs whatever are used. They are held open by a current that always flows through one winding on the cores. When the key is closed, a current twice as strong circulates in the opposite direction around the cores in a second and distinct winding. Hence, the magnetizing effect of the first current is not only neutralized, but the cores are magnetized just as strongly in the opposite direction. See Arts. **218** and **219.**

(68) Centering the polar armature, obtaining a resistance balance by means of the polar relay, obtaining a resistance balance by means of the neutral relay, obtaining a static balance, and adjusting the neutral relay for incoming signals.

(69) See Art. **274.**

(70) In the closed position of the transmitter in Fig. 75, the resistance in the circuit consists of 267 ohms in the coil C and a combined resistance of 900 ohms in the line and artificial line. This added to 267 ohms gives 1,167 ohms; hence, the current in the circuit will be $\frac{220}{1,167} = .18852$ ampere, or 188.52 milliamperes. Half of this, that is, 94.26 milliamperes, will flow through the line. When the transmitter is open, the coil C will be on open circuit and the current from the dynamo will divide at the point f, part going toward the relays and line and part through the coil A to the ground. The resistance of the circuit from the point f

to the ground now consists of three branches, one through A of 400 ohms, one through the line of 1,800 ohms, and one through the artificial line of 1,800 ohms. The combined resistance of these three paths will be $\frac{900 \times 400}{900 + 400} = 277$ ohms. This is in series with the coil B containing 800 ohms; hence, the total resistance of the circuit is $277 + 800 = 1{,}077$ ohms. The total current is $\frac{220}{1{,}077} = .20427$ ampere. This current will divide through the various branches that start at the point f inversely as their resistances. Hence, we have the proportion: the sum of the currents in the line and artificial-line circuits is to the total current (.20427), that is, the sum of the currents in all the branch circuits, as the combined resistance of all the branch circuits (277) is to the combined resistance of the line and artificial-line circuits (900). From this we get $.20427 \times \frac{277}{900} = .06287$ ampere, or 62.87 milliamperes. One-half of this, or 31.435 milliamperes, will flow through the line. From this it is evident that closing the transmitter increases the current in the line from 31.435 to 94.26 milliamperes, giving therefore a ratio of 1 to 3 almost exactly. See Art. **224.**

(71) See Art. **227.**

(72) See Art. **258.**

(73) To loose connections, broken wires, defective batteries, punctured condensers, defective resistance boxes, defective or improperly adjusted instruments, and the bad condition of the various contact points.

(74) When the transmitter (see Fig. 61) is open, the total resistance of the circuit with the values given in this question is $600 + 1{,}800 + \frac{900 \times 800}{900 + 800} = 2{,}823$ ohms. The total current is $\frac{220}{2{,}823} = .07793$ ampere. Since the line and artificial line whose combined resistance is 900 ohms is in

parallel with the 800 ohms in the leak coil, then the sum of the currents in the line and artificial line will be to the total current supplied by the dynamo (.07793) as the combined resistance of these three branch circuits $\left(\frac{800 \times 900}{800 + 900} = 423\right)$ is to the combined resistance of the line and artificial line (900). That is $x : .07793 = 423 : 900$; hence, the current in the line when the transmitter is open is $\frac{1}{2}\left(\frac{.07793 \times 423}{900}\right)$ $= .01831$ ampere. When the transmitter is closed, the total resistance of the circuit is $600 + 900 = 1{,}500$ ohms. Then, the total current is $\frac{220}{1{,}500} = .14667$ ampere. The current in the line when the transmitter is closed is $\frac{.14667}{2} = .07333$. Hence, the ratio of currents in the line in the open and closed position of the transmitter, 18.3 and 73.3 milliamperes, respectively, is very nearly 1 to 4.

(75) See Art. **260.**

(76) First suspect an improperly adjusted pole changer at the distant office; if that is not the cause, suspect the distant battery.

(77) (*a*) See Art. **271.**

(*b*) See Art. **272.**

(78) See Arts. **252** and **253.**

(79) (*a*) See Art. **123.**

(*b*) The tension of the spring must not be too great; the trunnion must not be too tight; and the local battery, and, consequently, the tension of the spring, must not be too weak. See Art. **122.**

(80) (*a*) See Art. **263.**

(*b*) See Art. **263.**

(81) (*a*) The station possessing the normal ground would find his apparatus out of balance. See Art. **276.**

(*b*) See Art. **276.**

(*c*) See Arts. **277** and **278.**

(82) See Art. **264.**

(83) It would be impossible to obtain a resistance balance. See Art. **275.**

TELEGRAPHY.

(PART 5.)

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(1) The polar relay of one set is arranged to control the pole changer of a second set, and the polar relay of the second set is arranged to control the pole changer of the first set. See Arts. **1** and **2** and Figs. 1 and 2.

(2) When dynamos are used, the proper current is obtained by inserting the proper amount of resistance (lamps or coils of German silver) in the circuit; but when primary cells are used, the proper current is obtained by using just enough cells.

(3) The repeating sounder on the common side of one set may be arranged to control the transmitter on the common side of the second set, and the repeating sounder of the second set, to control the transmitter of the first set. On the polar sides, the polar relay of the first set controls the pole changer of the second set, and, similarly, the polar relay of the second set controls the pole changer of the first set. Or, the repeating sounder on the common side of the first set may be arranged to control the pole changer of the second set, the polar relay of the second set controlling the transmitter of the first set; similarly, the repeating sounder on the common side of the second set controls the pole changer of the first set, and the polar relay of the first set controls the transmitter of the second set.

(4) The magnets in all local circuits are wound to 20 ohms, the resistance of all local circuits, including branch-office loops, are brought up to 100 ohms by means of the resistance coils and the local sounder, and the branch-office loop circuits are connected in parallel with the main-office circuits instead of in series, as is usually the case in other offices. In connection with each polar duplex or one-half of a quadruplex set, there are upon the desks two single-pole, double-throw, knife switches, which, in connection with the loop spring jacks and wedges, enable the chief operator to include a branch-office loop in the local circuit of a set or to connect the two sets so that they may repeat into one another. Each half of a quadruplex or repeater set is treated as a duplex set. See Arts. **5, 6,** and **7.**

(5) One advantage of this system over the Morse system is that it is less likely to be affected by ordinary trouble on the wire, and it will work readily across heavy escapes, and even when the wires are crossed or grounded, including the wire upon which the phonoplex is working. Furthermore, even bad weather fails to affect the signals to any great extent. A disadvantage of the system is the fact that only one phonoplex circuit can be worked successfully at the same time upon the same line of poles carrying a number of wires. A companion phonoplex on a line of poles on the opposite side of a railroad track may even be impracticable, for the reason that the phonoplex impulses are so penetrating that their inductive effects extend far into the space around the wire.

(6) A multiplex single-wire, or defective-loop, repeater is an arrangement of apparatus whereby a single branch line (usually a city branch-office line) may be so connected to a duplex or one side of a quadruplex set that the messages passing over the multiplex system may be sent over the branch line, and so that the branch office may send through the multiplex apparatus to the distant multiplex station. See Arts. **9** and **10.**

(7) The siphon is made to vibrate to and from the paper, in order to avoid the friction between the end of the siphon and the paper tape, which would impede the movement of the delicately suspended coil. Thus the siphon traces a dotted, instead of a continuous, line. The vibration of the siphon is also necessary because otherwise the ink is liable to gather upon the end of the siphon in globular form, and either blur the record or cause it to stop recording.

(8) The Hurd branch-office signaling device, as shown in Fig. 17, may be used. In this method an annunciator, connected to the branch-office wire, is so arranged that the shutter will fall and ring a bell if the branch-office operator momentarily grounds his line at the proper place by a switch provided and properly connected for this particular purpose.

(9) (*a*) Two line wires are used. The two line wires are used as a complete metallic circuit for telephonic communication and each wire with the earth as a return path constitutes one telegraph circuit. Thus the two line wires, with the earth as a common return path for both, provide two distinct and separate telegraph circuits.

(*b*) If only one wire is used, then one telegraph and one telephone message may be sent simultaneously. In this case, the earth is used as a common return path for both the telephone and telegraph currents.

(10) The siphon is made to vibrate by arranging the apparatus so that pulsatory currents are sent through an electromagnet. The frequency of these pulsations is made to coincide with the natural rate of vibration of the siphon. The lower end of the glass siphon has glued to it a minute piece of iron that the electromagnet, over which the paper tape passes, attracts every time a pulsatory current passes through the magnet. The pulsatory currents are caused by a reed and magnet, arranged in a manner similar to a vibrating bell. The rate of vibration of the reed is adjusted

to suit the natural rate of vibration of the siphon by regulating the height of a column of mercury in a glass tube attached to the reed. See Art. **87.**

(11) It is a combination of resistances and condensers so arranged that the artificial cable has not only the same resistance and capacity as the real cable it is intended to resemble, but it also charges and discharges at the same rate as the cable.

(12) (*a*) Two line wires are connected at each end through a high impedance coil. A telegraph set is connected between the center of each coil and the ground at each end, and the telephones at each end are connected directly across the two line wires. Hence, the two line wires form a complete metallic circuit for the telephones, and the impedance coils at each end prevent the passage of the telephone currents through them from one line to the other. The two line wires in parallel form one path (having only one-half the resistance of one line circuit), and the ground the other path for the telegraph current. At each end one-half of the impedance coil is in series with each line wire, and, hence, the two halves of the impedance coil at one end are in parallel with each other so far as the telegraph current is concerned, but in series with each other so far as the telephone current is concerned. Hence the impedance coil at each end offers only one-half the resistance of one-half of the whole coil (that is, one-fourth the resistance of the whole coil) to the telegraph current, and, moreover, the telegraph current does not produce any magnetism in the iron cores of these coils, because an equal current flows in each coil in opposite directions around the iron core. See Art. **65.**

(*b*) Two messages, one telephone message using the two line wires as a complete circuit, and one telegraph message using the two line wires in parallel as one path and the ground as the return path of the circuit. See Art. **64.**

(13) It is a multiplex single-wire, or defective-loop, repeater.

(14) The bridge duplex method. In the Muirhead double-block system, the resistances in two branch arms are replaced by condensers.

(15) (*a*) It is a multiplex single-wire, or defective-loop, repeater.

(*b*) It depends on the Toye-repeater principle, that is, on the substitution of a resistance in place of the branch-office line circuit when the repeating transmitter opens.

(16) A coil having considerable self-induction is included in the line circuit. Across the terminals of this coil is a local circuit including, say, merely a key and a battery. Whenever this local circuit is broken, the high self-induction of the magnetic coil sets up an impulse that travels over the line. This impulse causes the phone, which resembles a telephone receiver, to give out a sound resembling the click of a telegraph sounder. The closing of the local circuit does not affect the telephone; it is affected only by the opening of the local circuit. The noise produced by the down stroke of the sounder is imitated by breaking a strong current and the up stroke of the sounder is imitated by breaking a somewhat smaller current. See Arts. **72**, **73**, and **74**.

(17) (*a*) Not usually.

(*b*) Because the resistance of a main line is usually at least an appreciable part of the total resistance of the circuit; hence, any change in the line resistance due to a change in the weather would usually require a readjustment of the resistance in the Moffat repeater, or of the number of extra cells in the Downer repeater.

(18) The message is first punched in a paper tape by a machine called a perforator; the punched paper is then fed through a transmitter, and at the distant end is recorded in ink on a paper tape in the dot-and-dash characters of the regular telegraph code; an operator then translates the code record and writes the message on an ordinary message blank with or without a typewriter.

(19) (*a*) The accuracy and speed of working is very much better when automatic transmission is used in place of hand or manual transmission.

(*b*) The Cuttriss and the Crehore and Squier sine-wave transmitters.

(20) The Downer repeater depends on the principle of including an extra battery in the higher resistance circuit, so as to keep the current strong enough to hold closed the transmitter through the contacts of which the circuit that happens to be sending passes.

(21) The Half-Milliken repeater is used when it is necessary to connect a polar duplex or one side of a quadruplex set with a main line, so that the distant end of this main line may both send and receive (not simultaneously, however) through one wire from the repeater set.

(22) All the circuits are normally closed in the Half-Milliken repeater.

(23) By using condensers in the circuit at each end somewhere between the ground and the cable.

(24) By this repeater, a branch-office loop may be connected in circuit with a quadruplex set in such a manner that the two main terminal offices may work single and the branch office will be able to hear and to break, when necessary, either main terminal office that may be sending. The two ends can send double, that is, simultaneously, if it is not desired to send the messages to the branch offices. Furthermore, the branch office can send to the repeating office and to either main terminal office, in which case the line is worked single. See Arts. **26** to **29.**

(25) One key only need be placed between the ground and the two branch-office circuits. See Arts. **30, 31,** and **32** and Fig. 12.

(26) A double-loop repeater is an arrangement of apparatus that enables two branch offices to receive a message

that comes to one main office over a multiplex system from another main office, and it also allows either branch office to send to the other branch office and through the sending side of the multiplex set at the nearer main office to the distant main office.

(27) The key at station *A* must remain closed. Then, if *B* sends, the repeating sounder $R S_3$ will operate the pole changer of the set *C*; and if *C* sends, the repeating sounder $R S_4$ will operate the pole changer of the set *B*. Hence, messages may be both sent and received at both stations *B* and *C*. Confused signals, due to the sending at both *B* and *C*, will pass through the pole changer of the set *A*; hence, neither message can be read at *A*. See Arts. **38** to **41**.

(28) The contacts of the repeating transmitter may become dirty or corroded, the repeating transmitters may be improperly adjusted, and the current through the pole changer or transmitter magnet of the multiplex set may not be the same, as it should be, in the two positions of the repeating transmitter.

(29) This arrangement allows any one of the three stations to use the line as a simplex system, to send to the other two, and, moreover, allows any two to work double, that is, duplex, provided the key at the third station is kept closed.

TELEGRAPHY.

(PART 6.)

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(1) See Arts. **297** and **298.**

(2) It is a system developed by Crehore and Squier, in which an alternating sine-wave current is used. The circuit is never opened or closed except when the sine-wave curve is passing through its zero value. A telegraph code is arranged by holding the circuit open for the duration of one or more half or full current waves, and then closing the circuit for one or more half or full waves. See Arts. **172** to **175.**

(3) A perforated tape, the holes in which represent the message, is first prepared by means of three keys, one representing dots, the second spaces, and the third dashes. The depression of the dot key causes an electromagnet to punch a hole in the tape on one side of the center line and advances the tape a definite distance; the depression of the dash key produces a similar hole on the other side of the center line and advances the tape the same distance as the other key; the space key advances the tape the necessary distance between the letters. The tape is then fed through a transmitting device, the holes on one side causing positive-current impulses and the holes on the other side negative-current impulses to be sent over the line. At the receiver the dot, or positive, impulses produce a mark along the center of a chemically sensitized tape, while the dash, or negative, impulses produce two parallel marks along the tape, one on each side of the center line. These are translated and

written out on a message blank with or without a typewriter. The process is the same as in the Wheatstone automatic system, but it is capable of transmitting messages at a very much higher speed. See Arts. **160** to **166.**

(4) (*a*) The Thomson, or reflecting, galvanometer and the D'Arsonval galvanometer.

(*b*) The Thomson galvanometer is by far the most sensitive and, therefore, suitable where tests of the greatest accuracy are to be made. It, however, has the disadvantage of being affected by slight external magnetic disturbances. The D'Arsonval galvanometer is the most satisfactory instrument for general use. It is sufficiently sensitive for all practical purposes, and is not affected by external magnetic fields. The suspension of the needle is not so delicate as in the Thomson, thus making it easier to set up and less liable to injury.

(5) One serious objection charged against wireless telegraphy is the fact that with the method generally used at present, two independent communications cannot be received at the same station readily, if at all; and every receiver placed within the radius of action of a transmitter is acted upon by the waves sent out by the one transmitter. Hence, if two transmitters are simultaneously operated, complete interference or a confusion of the two sets of signals is the result. A second serious objection or defect is the comparatively short distance over which it may be worked. Up to March, 1901, the longest distance across which telegraph signals had been successfully sent was 200 miles over water and much less over land. Both of these objections may, of course, be more or less overcome in the future.

(6) They enable the sending over one wire of from 3 to 20 times as many messages as could be sent in the same time over a simple circuit by hand. On the other hand, they are more complicated than the Morse key-and-sounder method, and the various steps through which the message passes increase the probability of making mistakes.

(7) A coherer is a device for detecting the presence of electromagnetic waves. It is sensitive to electromagnetic waves, because the latter cause the resistance of the coherer to decrease enormously.

(8) A perforated tape representing the message must be prepared; this tape must then be fed through the transmitter. In the first method described, the signals are received in zigzag lines on sensitive photographic paper. The photographic paper must be developed and then the zigzag code message translated and written on a message blank. In the writing telegraph system, the message is received in a rather stilted form of writing on a sensitized sheet of photographic paper that must be developed.

(9) The objections are about the same as to all automatic telegraph systems, namely, the complication of the apparatus and the several steps through which the message must pass, and there is also the serious objection to the additional complication rendered necessary by the use of the sensitized photographic paper.

(10) See Arts. **280** and **281**.

(11) A deflection is first obtained through a known resistance with a given battery, and from this the constant of the galvanometer is computed. After this, the deflection produced by a current from the same battery through the insulation resistance to be measured is noted, and by comparing this deflection with that through the known resistance, due allowance being made for the galvanometer shunts that are used, the insulation resistance may be computed. See Arts. **287** to **290**.

(12) See Art. **306**.

(13) (*a*) See Art. **274**.
(*b*) See Art. **275**.

(14) $342 \times 1{,}000 \times \frac{1}{10} = 34{,}200$.

(15) The Wheatstone bridge.

(16) See Art. **290.**

(17) Because the electrostatic capacity of long circuits is frequently high enough to allow enough current to pass into and out of the circuit to ring the polarized bell of the testing set, thus producing the same effect as if the circuit were continuous, or as if a ground or cross existed.

(18) By substituting in formula **17,** in which $R = 515.58$, $m = 1,000$, $n = 1,000$, and $p = 2,015$ ohms, we get as the resistance along the bad line to the fault

$$x = \frac{1,000 \times 515.58}{1,000 + 1,000 + 2,015} = 128.4 \text{ ohms.}$$

Then, the distance to the fault in feet from the testing station equals

$$\frac{128.4 \times 1,000}{2.578} = 49,806 \text{ ft., or } 9.43 \text{ mi. Ans.}$$

(19) Call the first measurement, which gave 1,077 ohms, a; the second 1,130 ohms, b, and the third 1,184 ohms, c; then, substituting these values in formulas **4, 5,** and **6,** we get

$$\text{Resistance of line } x = \frac{1,077 + 1,130 - 1,184}{2} = 511.5 \text{ ohms.}$$

$$\text{Resistance of line } y = \frac{1,077 + 1,184 - 1,130}{2} = 565.5 \text{ ohms.}$$

$$\text{Resistance of line } z = \frac{1,130 + 1,184 - 1,077}{2} = 618.5 \text{ ohms.}$$

Ans.

(20) The insulation resistance per mile is found by multiplying the total insulation resistance found from the measurement by the length of the line in miles, or by multiplying by the length of the line in feet and dividing this result by 5,280 (the number of feet in 1 mile).

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